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Travelling Wave Solutions of Cahn-Allen Equation by the (G'/G)-expansion Method

(G'/G)-展开方法求 Cahn-Allen 方程的行波解

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Abstract: Travelling wave solutions of the Cahn-Allen equation are established by (G'/G)-expansion method. The obtained results include periodic and solitary wave solutions. The power of this manageable method is confirmed.

Key words: Cahn-Allen equation, exact solutions, travelling wave solutions, (G'/G)-expansion method

摘要:应用 (G'/G)- 展开方法导出 Cahn-Allen 非线性方程的行波解. 该方法简单、可行,而且所得结果包含周期解和孤波解.

关键词:Cahn-Allen 方程 精确解 行波解 (G'/G)-展开方法
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The investigation of the traveling wave solutions for nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. During the past several decades, the investigation of new exact solutions have received considerable attention. A variety of powerful methods, such as inverse scattering method[1], bilinear transformation[2], extended tanh method[3], Exp-function method[4] and first integral method[5] were used to develop nonlinear dispersive and dissipative problems.

The pioneer Wang[6] introduced the (G'/G)-expansion method for a reliable treatment of the nonlinear wave equations. The useful (G'/G)-expansion method is widely used by many such as in reference [7] and the references cited therein.

Our first interest in the present paper is in implementing the (G'/G)-expansion method to stress its power in handling nonlinear equations, so that one can apply it to models of various types of nonlinearity. The rest of the paper is organized as follows. Then, we illustrated this method in detail with the celebrated the Cahn-Allen equation, and some conclusions are given.

We consider the Cahn-Allen equation

$$u_t = u_{xx} - u^3 + u, \tag{1}$$

which arise in many scientific applications such as mathematical biology, quantum mechanics and plasma physics. Using the wave variable $\zeta = x - ct$, the equation (1) is carried to a ODE

$$u'' - u^3 + u + cu' = 0, \tag{2}$$

where the prime denote the derivation with respect to ζ.

Balancing u'' with u^3 in equation (2) gives

$$m + 2 = 3m, \tag{3}$$

so that $m = 1$. Suppose that the solutions of equation (2) can be expressed by a polynomial in (G'/G) as follows:

$$u(\zeta) = a_0 + a_1(G'/G), a_1 \neq 0, \tag{4}$$

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where $G(\zeta)$ satisfies the second order ODE in the form

$$\frac{d^2G(\zeta)}{d\zeta^2}+\lambda\frac{dG(\zeta)}{d\zeta}+\mu G(\zeta)=0. \quad (5)$$

By using equation (5), from equation (4) we have

$$u^3(\zeta)=a_0^3+3a_0^2a_1(G'/G)+3a_0a_1^2(G'/G)^2+a_1^3(G'/G)^3, \quad (6)$$

$$u'(\zeta)=-a_1[\mu+\lambda(G'/G)+(G'/G)^2], \quad (7)$$

$$u''(\zeta)=a_1\lambda\mu+(a_1\lambda^2+2a_1\mu)(G'/G)+3a_1\lambda(\frac{G'}{G})^2+2a_1(G'/G)^3. \quad (8)$$

Substituting equations (6) ~ (8) into equation (2), collecting the coefficients of $(G'/G)^i, i=0,\cdots,3$ and set it to zero we obtain the system

$$\begin{aligned}&2a_1-a_1^3=0,\\&3a_1\lambda-3a_0a_1^2-ca_1=0,\\&a_1\lambda^2+2a_1\mu-3a_0^2a_1+a_1-ca_1\lambda=0,\\&a_1\lambda\mu-a_0^3+a_0-ca_1\mu=0.\end{aligned} \quad (9)$$

Solving this system by Maple gives

$$a_1=\pm\sqrt{2},\lambda=\pm\frac{2a_0-1}{\sqrt{2}},c=\mp\frac{3}{\sqrt{2}},\mu=\frac{(a_0-1)a_0}{2},$$

$$a_1=\pm\sqrt{2},\lambda=\pm\frac{2a_0}{\sqrt{2}},c=0,\mu=\frac{(a_0-1)(a_0+1)}{2},$$

$$a_1=\pm\sqrt{2},\lambda=\pm\frac{2a_0+1}{\sqrt{2}},c=\pm\frac{3}{\sqrt{2}},\mu=\frac{(a_0+1)a_0}{2}, \quad (10)$$

where a_0 is an arbitrary constant.

Substituting equation (10) into equation (4) yields

$$u(\zeta)=a_0\pm\sqrt{2}(G'/G), \quad (11)$$

where $\zeta=x\pm\frac{3t}{\sqrt{2}},\zeta=x$, or $\zeta=x\mp\frac{3t}{\sqrt{2}}$.

Substituting general solutions of equation (5) into equation (11) we have six types of travelling wave solutions of the Cahn-Allen equation as follows:

$$u_{1,2}(x,t)=\frac{1}{2}[1+\tanh(\pm\frac{x}{2\sqrt{2}}+\frac{3t}{4})],$$

$$u_{3,4}(x,t)=\frac{1-c_0\cdot\exp(\mp\sqrt{2}x)}{1+c_0\cdot\exp(\mp\sqrt{2}x)},u_{5,6}(x,t)=\frac{-1}{1+c_0\cdot\exp(\pm\frac{x}{\sqrt{2}}-\frac{3t}{2})}$$

In particular, if $c_0=1$, then $u_{3,4}(x,t)$ and $u_{5,6}(x,t)$ become

$$u_3(x,t)=\tanh(\frac{x}{\sqrt{2}}),u_4(x,t)=-\tanh(\frac{x}{\sqrt{2}}),u_5(x,t)=-\frac{1}{2}[1-\tanh(\frac{x}{2\sqrt{2}}-\frac{3t}{4})],u_6(x,t)=-\frac{1}{2}[1+\tanh(\frac{x}{2\sqrt{2}}+\frac{3t}{4}].$$

If $c_0=-1$, then $u_{3,4}(x,t)$ and $u_{5,6}(x,t)$ become

$$u_3(x,t)=\coth(\frac{x}{\sqrt{2}}),u_4(x,t)=-\coth(\frac{x}{\sqrt{2}}),u_5(x,t)=-\frac{1}{2}[1-\coth(\frac{x}{2\sqrt{2}}-\frac{3t}{4})],u_6(x,t)=-\frac{1}{2}[1+\coth(\frac{x}{2\sqrt{2}}+\frac{3t}{4})].$$

Comparing our results and Wazwaz's results[8] then it can be seen that the results are the same.

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