第 18 卷第 3 期　　广西科学院学报　　Vol. 18, No. 3
2002 年 8 月　　Journal of Guangxi Academy of Sciences　　August 2002

一种求 Ramsey 数的命题演算法*
A Propositional Calculus Method to Find Ramsey Numbers

罗示丰
Luo Shifeng

（广西大学计算机与信息工程学院　南宁　530004）
(Computer & Information Engineering College, Guangxi University, Nanning, 530004)

摘要　首先在矩阵中用谓词描述"含 K_n"和"含 $\overline{K}_m$"，然后通过命题演算法逐行求出不含 K_n 也不含 $\overline{K}_m$ 的矩阵 $(a_{ij})_{j=i+1,\dots,l}^{i=1,2,\dots,l-1}$. 若对于 $l \leqslant r-1$ 有这样的矩阵，而对于 $l=r$ 却没有，则 r 为 Ramsey 数．

关键词　Ramsey 数　命题演算　三角矩阵
中图法分类号　O141

Abstract　The "include K_n and $\overline{K}_m$" in a matrix are firstly described by predicate. Then the matrix $(a_{ij})_{j=i+1,\dots,l}^{i=1,2,\dots,l-1}$ that include neither K_n, nor $\overline{K}_m$, is found by a prepositional calculus in row by row. If there are the matrixes for $l \leqslant r-1$, but there is no the matrix for $l=r$, then r is Ramsey number.

Key words　Ramsey number, prepositional calculus, triangle matrix

目前对 Ramsey 数的研究正如火如荼．由于 Ramsey 数计算的困难，学者们纷纷转向 Ramsey 数的界的研究[1]．本人认为，更有效的计算方法和更接近的界，两者不可偏废．为此，本文提出一种全新的、较为简捷的计算方法，有助于求出未知的 Ramsey 数．

1　主要定理及其证明

1.1　基本定理

定理 1　一个简单无向连通图 $G=(V,E)$ 是完全图的充要条件是

$$VC(G)=\{V_{v_1}^{\{v_2,v_3,\cdots,v_n\}},V_{v_2}^{\{v_3,v_4,\cdots,v_n\}},\cdots,V_{v_{n-1}}^{\{v_n\}}\}^{[2]}. \tag{1}$$

证明　设 G 是具有 n 个顶点的完全图．我们按 VC 算法[2]依次删去 $v_1,v_2,\cdots,v_{n-1}$ 各点及其关联的边，就是

$V_{v_1}^{\{v_2,v_3,\cdots,v_n\}},V_{v_2}^{\{v_3,v_4,\cdots,v_n\}},\cdots,V_{v_{n-1}}^{\{v_n\}}$ 于是就有上述的 $VC(G)$ 式．

如果某个简单无向连通图实施 VC 算法，其 $VC(G)$ 形如(1)式，则以

2002-02-25 收稿，2002-04-10 修回．
*广西大学科研基金资助项目．

$V = \{v_1, v_2, \cdots, v_n\}, E = \{(v_1,v_2),(v_1,v_3),\cdots,(v_1,v_n),(v_2,v_3),\cdots,(v_2,v_n),\cdots,(v_{n-1},v_n)\}$ 的图 $G = (V,E)$ 因其 $|E| = n(n-1)/2(|V| = n)$ 而为完全图．

1.2 简单连通图 $G=(V,E)$ 含有 K_n 的表述

任一个简单无向连通图 $G = (V,E)$，其邻接矩阵可由一个三角矩阵

$$\begin{bmatrix} a_{12} & a_{13} & \cdots & a_{1r} \\ & a_{23} & \cdots & a_{2r} \\ & & \ddots & \vdots \\ & & & a_{(r-1)r} \end{bmatrix} \left[|V| = r, a_{ij} = \begin{cases} 1,(v_i,v_j) \in E \\ 0,(v_i,v_j) \notin E \end{cases} \begin{matrix} i = 1,2,\cdots,r-1 \\ j = i+1,\cdots,r \end{matrix} \right],$$

表示．$G = (V,E)$ 含有 K_m 意即在其三角矩阵中

$$\exists r_1 \exists r_2 \cdots \exists r_m (1 \leqslant r_m < r_{m-1} < \cdots < r_2 < r_1 \leqslant r \wedge a(r_2,r_1) = 1 \wedge a(r_3,r_2) = 1 \wedge a(r_3,r_1) = 1 \wedge \cdots \wedge a(r_m,r_{m-1}) = 1 \wedge a(r_m,r_{m-2}) = 1 \wedge \cdots \wedge a(r_m,r_1) = 1), \tag{2}$$

此表达式直接由(1) 式而得，只需把 r_i 视为 v_i 即可．如果我们注意到原三角矩阵中 $0-1$ 互易所成的矩阵就是原图的补图的三角矩阵，那么，在(2) 式中所有的 1 都改为 0，m 改为 n，即

$$\exists r_1 \exists r_2 \cdots \exists r_n (1 \leqslant r_n < r_{n-1} < \cdots < r_2 < r_1 \leqslant r \wedge a(r_2,r_1) = 0 \wedge a(r_3,r_2) = 0 \wedge a(r_3,r_1) = 0 \wedge \cdots \wedge a(r_n,r_{n-1}) = 0 \wedge a(r_n r_{n-2}) = 0 \wedge \cdots \wedge a(r_n,r_1) = 0) \tag{3}$$

就体现了图 $G = (V,E)$) 含有 $\overline{K}_n$

2 如何判断 1 个数 r 是否是 Ramsey 数

据定义[3]，任意一个有 r 个顶点的图，如它不含 K_m 就必含 $\overline{K}_n$，这样的最小数 r，就是所谓的 Ramsey 数，表示为 $R(K_m,\overline{K}_n) = r$. 其中 K_m 表示 m 个顶点的完全图，$\overline{G}$ 表示 G 的补图．

由此定义，如有 l 个顶点的图，它既不含 K_m 也不含 $\overline{K}_n$，那么，l 就不是关于 K_m 和 $\overline{K}_n$ 的 Ramsey 数；倘若 $(r-1)$ 个顶点的图，不含 K_m 也不含 $\overline{K}_n$，而对于 r 个顶点，就不再是“不含 K_m 也不含 $\overline{K}_n$”，那么这个 r 就是 Ramsey 数 $(R(K_m,\overline{K}_n) = r)$.

3 用于推演的 2 个谓词公式

为简便，用 P_{ij} 和 $\overline{P}_{ij}$(命题 P_{ij} 的否定) 分别表示 $a(i,j) = 1$ 和 $a(i,j) = 0$，而且把 $P \wedge Q$ 略为 PQ. 如此(2) 式则写成

$$\exists r_1 \exists r_2 \cdots \exists r_m (P_{r_2 r_1} P_{r_3 r_2} P_{r_3 r_1} \cdots P_{r_m r_{m-1}} P_{r_m r_{m-2}} \cdots P_{r_m} P_{r_1}),$$

取上述谓词公式的否定(不含 K_m) 并用有关谓词演算的等价式[4] 可得

$$\forall_{r_1} \forall_{r_2} \cdots \forall_{r_m} (\overline{P_{r_2 r_1} P_{r_3 r_2} P_{r_3 r_1} \cdots P_{r_{m-1} r_m} P_{r_{m-1} r_{m-3}} \cdots P_{r_{m-1} r_1}} \vee \overline{P_{r_m r_{m-1}} \cdots P_{r_m r_{m-1}}}),$$

其中 $1 \leqslant r_m < r_{m-1} < \cdots < r_2 < r_1 \leqslant r$，即

$$\forall_{r_1} \forall_{r_2} \cdots \forall_{r_m} (P_{r_2 r_1} P_{r_3 r_2} P_{r_3 r_1} \cdots P_{r_{m-1} r_{m-2}} P_{r_{m-1} r_{m-3}} \cdots P_{r_{m-1} r_1} \rightarrow \overline{P_{r_m r_{m-1}} \cdots P_{r_m r_1}}), \tag{4}$$

特别当 $m = 3, r_2 = i, r_1 = j$ 和 $r_3 = k$ 时(4) 式成为

$$\forall_i \forall_j \forall_k (P_{ij} \rightarrow \overline{P_{ki} P_{kj}}) \Leftrightarrow \forall_i \forall_j \forall_k (P_{ij} \rightarrow \overline{P}_{ki} \vee \overline{P}_{kj})(1 \leqslant k < i < j), \tag{4'}$$

同理可得(不含 $\overline{K}_n$)

$$\forall_{r_1} \forall_{r_2} \cdots \forall_{r_n} (\overline{P}_{r_2 r_1} \overline{P}_{r_3 r_2} \cdots \overline{P}_{r_{n-1} r_{n-2}} \overline{P}_{r_{n-1} r_{n-3}} \cdots \overline{P}_{r_{n-1} r_1} \rightarrow \overline{\overline{P}_{r_n r_{n-1}} \cdots \overline{P}_{r_n r_1}}), \tag{5}$$

其中 $1 \leqslant r_n < r_{n-1} < \cdots < r_2 < r_1 \leqslant r$ 以及$(n = 3)$，

$$\forall_i \forall_j \forall_k (P_{ij} \rightarrow \overline{\overline{P}_{ki}\overline{P}_{kj}}) \Leftrightarrow (\overline{P}_{ij} \rightarrow P_{ki} \vee P_{kj}), (1 \leqslant k < i < j), \qquad (5')$$

(4)、(5) 式就是下面的推演所常用的谓词公式．现以 $R(K_3,\overline{K}_3) = 6$ 的证明为例说明．

3.1 $R(K_3,\overline{K}_3) \neq 4$ 的证明

P_{34}(对于 $\overline{P}_{34}$,其推演类似)$\Rightarrow \overline{P}_{23} \vee \overline{P}_{24} \Leftrightarrow \overline{P}_{23}P_{24} \vee P_{23}\overline{P}_{24} \vee \overline{P}_{23}\overline{P}_{24}$(由(4′)式并化主范式)
$P_{23}\overline{P}_{24} \Rightarrow (\overline{P}_{12} \vee P_{13})(P_{12} \vee P_{14}) \Leftrightarrow P_{12}\overline{P}_{13} \vee \overline{P}_{12}P_{14} \vee P_{13}P_{14} \Rightarrow (P_{12}\overline{P}_{13} \vee \overline{P}_{12}P_{14} \vee \overline{P}_{13}P_{14})(\overline{P}_{13} \vee \overline{P}_{14}) \Leftrightarrow P_{12}\overline{P}_{13} \vee \overline{P}_{12}P_{13}P_{14} \vee P_{12}\overline{P}_{13}\overline{P}_{14} \vee \overline{P}_{13}P_{14} \Leftrightarrow P_{12}\overline{P}_{13}P_{14} \vee P_{12}\overline{P}_{13}\overline{P}_{14} \vee \overline{P}_{12}\overline{P}_{13}P_{14}$(主范式).

因此,得到的三角矩阵及其对应图如图1所示．

我们找到了不含 K_3 也不含 $\overline{K}_3$ 的4个顶点的图,故 $R(K_3,\overline{K}_3) \neq 4$.

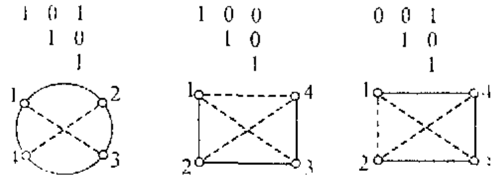

图1 三角矩阵及其对应图

3.2 $R(K_3,\overline{K}_3) \neq 5$ 的证明

$P_{45} \Rightarrow \overline{P_{34}P_{35}} \Leftrightarrow \overline{P}_{34} \vee \overline{P}_{35} \Leftrightarrow \overline{P}_{34}(P_{35} \vee \overline{P}_{35}) \vee \overline{P}_{35}(P_{34} \vee \overline{P}_{34}) \Leftrightarrow P_{34}\overline{P}_{35} \vee \overline{P}_{34}P_{35} \vee \overline{P}_{34}\overline{P}_{35}$
$P_{34}\overline{P}_{35} \Rightarrow (P_{23} \vee \overline{P}_{24})(\overline{P}_{23} \vee \overline{P}_{25}) \Rightarrow (\overline{P}_{23}P_{25} \vee P_{23}\overline{P}_{24} \vee \overline{P}_{24}P_{25})(\overline{P}_{24} \vee \overline{P}_{25})$(注:由(4′)和(5′)式得)
$\Leftrightarrow \overline{P}_{23}\overline{P}_{24}P_{25} \vee P_{23}\overline{P}_{24} \vee \overline{P}_{24}P_{25} \vee P_{23}\overline{P}_{24}\overline{P}_{25}$
$\Leftrightarrow \overline{P}_{23}\overline{P}_{24}P_{25} \vee P_{23}\overline{P}_{24}(P_{25} \vee \overline{P}_{25}) \vee \overline{P}_{24}P_{25}(P_{23} \vee \overline{P}_{23}) \vee P_{23}\overline{P}_{24}\overline{P}_{25}$
$\Leftrightarrow P_{23}\overline{P}_{24}\overline{P}_{25} \vee P_{23}\overline{P}_{24}P_{25} \vee \overline{P}_{23}\overline{P}_{24}P_{25}$
$P_{23}\overline{P}_{24}\overline{P}_{25} \Rightarrow (\overline{P}_{12} \vee P_{13})(P_{12} \vee P_{14})(P_{12} \vee P_{15}) \Leftrightarrow (\overline{P}_{12} \vee \overline{P}_{13})(P_{12} \vee P_{14}P_{15})$
$\Leftrightarrow P_{12}\overline{P}_{13} \vee \overline{P}_{12}P_{14}P_{15} \vee \overline{P}_{13}P_{14}P_{15}$
$\Leftrightarrow (P_{12}\overline{P}_{13} \vee \overline{P}_{12}P_{14}P_{15} \vee \overline{P}_{13}P_{14}P_{15})(\overline{P}_{13} \vee \overline{P}_{14})(P_{13} \vee P_{15})(\overline{P}_{14} \vee \overline{P}_{15})$(注:由(4′)和(5′)得)
$\Leftrightarrow (P_{12}\overline{P}_{13} \vee P_{12}P_{14}P_{15} \vee \overline{P}_{13}P_{14}P_{15})(P_{13}\overline{P}_{14} \vee \overline{P}_{14}P_{15} \vee \overline{P}_{13}P_{15})(\overline{P}_{14} \vee \overline{P}_{15})$
$\Leftrightarrow (P_{12}\overline{P}_{13} \vee \overline{P}_{12}P_{14}P_{15} \vee \overline{P}_{13}P_{14}P_{15})(P_{13}\overline{P}_{14} \vee \overline{P}_{14}P_{15} \vee \overline{P}_{13}\overline{P}_{14}P_{15} \vee P_{13}\overline{P}_{14}\overline{P}_{15})$
$\Leftrightarrow P_{12}\overline{P}_{13}\overline{P}_{14}P_{15}$(注:从 $\overline{P}_{45}$ 出发所作的推演与此类似).

从而得到一个三角矩阵及其对应图如图2所示．这也是一个不含 K_3 也不含 $\overline{K}_3$ 的5个顶点的图．
$\therefore R(K_3,\overline{K}_3) \neq 5$.

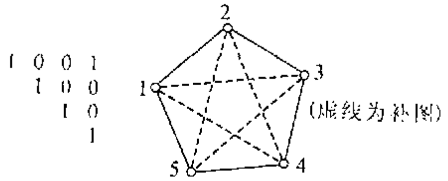

图2 三角矩阵及其对应图

3.3 不存在6个顶点,不含 K_3 也不含 $\overline{K}_3$ 的图的证明

证明

1)$P_{56} \Rightarrow \overline{P_{45}P_{46}} \Leftrightarrow (\overline{P}_{45} \vee \overline{P}_{46}) \Leftrightarrow \overline{P}_{45}P_{46} \vee P_{45}\overline{P}_{46} \vee \overline{P}_{45}\overline{P}_{46}$.

$\overline{P}_{45}P_{46} \Rightarrow \overline{\overline{P}_{34}\overline{P}_{35}}\,\overline{P_{34}P_{36}} \Leftrightarrow (P_{34} \vee P_{35})(\overline{P}_{34} \vee \overline{P}_{36}) \Leftrightarrow P_{34}\overline{P}_{36} \vee \overline{P}_{34}P_{35} \vee P_{35}\overline{P}_{36}$
$\Rightarrow (P_{34}\overline{P}_{36} \vee \overline{P}_{34}P_{35} \vee P_{35}\overline{P}_{36})(\overline{P}_{35} \vee \overline{P}_{36})$(注:由(4′)和(5′)式得)
$\Leftrightarrow P_{34}\overline{P}_{35}\overline{P}_{36} \vee P_{34}\overline{P}_{36} \vee \overline{P}_{34}P_{35}\overline{P}_{36} \vee P_{35}\overline{P}_{36} \Leftrightarrow P_{34}P_{35}\overline{P}_{36} \vee \overline{P}_{34}P_{35}\overline{P}_{36} \vee P_{34}\overline{P}_{35}\overline{P}_{36}$,
$P_{34}P_{35}\overline{P}_{36} \Rightarrow (\overline{P}_{23} \vee \overline{P}_{24})(\overline{P}_{23} \vee \overline{P}_{25})(P_{23} \vee P_{26}) \Leftrightarrow \overline{P}_{23}P_{26} \vee P_{23}\overline{P}_{24}\overline{P}_{25} \vee \overline{P}_{24}\overline{P}_{25}P_{26}$
$\Rightarrow (\overline{P}_{23}P_{26} \vee P_{23}\overline{P}_{24}\overline{P}_{25} \vee \overline{P}_{24}\overline{P}_{25}P_{26})(P_{24} \vee P_{25})(\overline{P}_{24} \vee \overline{P}_{26})(\overline{P}_{25} \vee \overline{P}_{26})$(注:由(4′)和(5′)式得)
$\Leftrightarrow (\overline{P}_{23}P_{26} \vee P_{23}\overline{P}_{24}\overline{P}_{25} \vee \overline{P}_{24}\overline{P}_{25}P_{26})(P_{24}\overline{P}_{25}\overline{P}_{26} \vee P_{24}\overline{P}_{26} \vee \overline{P}_{24}P_{25}\overline{P}_{26} \vee P_{25}\overline{P}_{26}) \Leftrightarrow F(F = FALSE)$,
$\overline{P}_{34}P_{35}\overline{P}_{36} \Rightarrow (P_{23} \vee P_{24})(\overline{P}_{23} \vee \overline{P}_{25})(P_{23} \vee P_{26}) \rightarrow (P_{23}\overline{P}_{25} \vee \overline{P}_{23}P_{24}P_{26} \vee P_{24}\overline{P}_{25}P_{26})(P_{24} \vee P_{25})(\overline{P}_{24} \vee \overline{P}_{26})(\overline{P}_{25} \vee P_{26}) \Leftrightarrow (P_{23}\overline{P}_{25} \vee \overline{P}_{23}P_{24}P_{26} \vee P_{24}P_{25}P_{26})(P_{24}\overline{P}_{25}\overline{P}_{26} \vee P_{24}\overline{P}_{26} \vee \overline{P}_{24}P_{25}\overline{P}_{26} \vee P_{25}\overline{P}_{26}) \Leftrightarrow P_{23}P_{24}\overline{P}_{25}\overline{P}_{26}$
$\Rightarrow (\overline{P}_{12} \vee \overline{P}_{13})(\overline{P}_{12} \vee \overline{P}_{14})(P_{12} \vee P_{15})(P_{12} \vee P_{16}) \Leftrightarrow \overline{P}_{12}P_{15}P_{16} \vee P_{12}\overline{P}_{13}\overline{P}_{14} \vee \overline{P}_{13}\overline{P}_{14}P_{15}P_{16}$
$\Rightarrow (\overline{P}_{12}P_{15}P_{16} \vee P_{12}\overline{P}_{13}\overline{P}_{14} \vee \overline{P}_{13}\overline{P}_{14}P_{15}P_{16})(P_{13} \vee P_{14})(\overline{P}_{13} \vee \overline{P}_{15})(P_{13} \vee P_{16})(P_{14} \vee P_{15})(\overline{P}_{14} \vee \overline{P}_{16})(\overline{P}_{15} \vee \overline{P}_{16}) \Leftrightarrow (P_{12}\overline{P}_{13}\overline{P}_{14} \vee (\overline{P}_{12} \vee \overline{P}_{13}\overline{P}_{14})P_{15}P_{16})(P_{13}\overline{P}_{15} \vee \overline{P}_{13}P_{14}P_{16} \vee P_{14}\overline{P}_{15}P_{16} \vee P_{14}\overline{P}_{15}\overline{P}_{16} \vee P_{14}P_{15}\overline{P}_{16} \vee P_{15}\overline{P}_{16}$

$\vee P_{14}\bar{P}_{16})\Leftrightarrow\bar{P}_{12}\bar{P}_{13}P_{14}P_{15}P_{16}(P_{14}\bar{P}_{15}\bar{P}_{16}\vee\bar{P}_{14}P_{15}\bar{P}_{16}\vee P_{15}\bar{P}_{16}\vee P_{14}\bar{P}_{16})\Leftrightarrow F$，

$P_{34}\bar{P}_{35}\bar{P}_{36}\Rightarrow(\bar{P}_{23}\vee\bar{P}_{24})(P_{23}\vee P_{25})(P_{23}\vee P_{26})\Leftrightarrow P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee\bar{P}_{24}P_{25}P_{26}$

$\Rightarrow(P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee P_{24}\bar{P}_{25}P_{26})(P_{24}\vee P_{25})(\bar{P}_{24}\vee\bar{P}_{26})(\bar{P}_{25}\vee\bar{P}_{26})$

$\Leftrightarrow(P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee P_{24}\bar{P}_{25}P_{26})(P_{24}\bar{P}_{25}\bar{P}_{25}\vee P_{24}\bar{P}_{26}\vee\bar{P}_{24}P_{25}\bar{P}_{26}\vee P_{25}\bar{P}_{26})\Leftrightarrow P_{23}\bar{P}_{24}P_{25}\bar{P}_{26}$

$\rightarrow(\bar{P}_{12}\vee\bar{P}_{13})(P_{13}\vee P_{14})(\bar{P}_{12}\vee\bar{P}_{15})(P_{12}\vee P_{16})(\bar{P}_{13}\vee\bar{P}_{14})(P_{13}\vee P_{15})(P_{13}\vee P_{15})(P_{14}\vee P_{15})(\bar{P}_{14}\vee$
$\bar{P}_{15})(\bar{P}_{15}\vee\bar{P}_{16})\Leftrightarrow(P_{12}\bar{P}_{13}\bar{P}_{15}\vee\bar{P}_{12}P_{14}P_{16}\vee\bar{P}_{13}P_{14}\bar{P}_{15}P_{16})P_{13}\bar{P}_{14}P_{15}\bar{P}_{16}\Leftrightarrow F$，

2) $P_{56}\Rightarrow\overline{P_{45}P_{46}}\Leftrightarrow\bar{P}_{45}\vee\bar{P}_{46}\Leftrightarrow\bar{P}_{45}P_{46}\vee P_{45}\bar{P}_{46}\vee\bar{P}_{45}\bar{P}_{46}$.

$P_{45}\bar{P}_{46}\Rightarrow\overline{P_{34}P_{35}}\,\overline{\bar{P}_{34}\bar{P}_{36}}\Leftrightarrow(\bar{P}_{34}\vee\bar{P}_{35})(P_{34}\vee P_{36})\Leftrightarrow\bar{P}_{34}P_{36}\vee P_{34}\bar{P}_{35}\vee\bar{P}_{35}P_{36}\Rightarrow(\bar{P}_{34}P_{36}\vee P_{34}\bar{P}_{35}\vee\bar{P}_{35}P_{36})(\bar{P}_{35}$
$\vee\bar{P}_{36})$（由(4′)式）

$\Leftrightarrow\bar{P}_{34}\bar{P}_{35}P_{36}\vee P_{34}\bar{P}_{35}\vee\bar{P}_{35}P_{36}\vee P_{34}\bar{P}_{35}\bar{P}_{36}\Leftrightarrow P_{34}\bar{P}_{35}P_{36}\vee P_{34}\bar{P}_{35}\bar{P}_{36}\vee\bar{P}_{34}\bar{P}_{35}P_{36}$

$P_{34}\bar{P}_{35}P_{36}\Rightarrow(\bar{P}_{23}\vee\bar{P}_{24})(P_{23}\vee P_{25})(\bar{P}_{23}\vee\bar{P}_{26})\Leftrightarrow\bar{P}_{23}P_{25}\vee P_{23}\bar{P}_{24}\bar{P}_{26}\vee\bar{P}_{24}P_{25}\bar{P}_{26}$

$\Rightarrow(\bar{P}_{23}P_{25}\vee P_{23}\bar{P}_{24}\bar{P}_{26}\vee\bar{P}_{24}P_{25}\bar{P}_{26})(\bar{P}_{24}\vee\bar{P}_{25})(P_{24}\vee P_{26})(\bar{P}_{25}\vee\bar{P}_{26})$（注：由(5′)和(4′)式得）

$\Leftrightarrow(\bar{P}_{23}P_{25}\vee P_{23}\bar{P}_{24}\bar{P}_{26}\vee\bar{P}_{24}P_{25}\bar{P}_{26})(\bar{P}_{24}\bar{P}_{25}P_{26}\vee P_{24}\bar{P}_{25}\vee\bar{P}_{25}P_{26}\vee P_{24}\bar{P}_{25}\bar{P}_{26})\Leftrightarrow F$，

$P_{34}\bar{P}_{35}\bar{P}_{36}\Rightarrow(\bar{P}_{23}\vee\bar{P}_{24})(P_{23}\vee P_{25})(P_{23}\vee P_{26})\Leftrightarrow P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee\bar{P}_{24}P_{25}P_{26}$

$\Rightarrow(P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee\bar{P}_{24}P_{25}P_{26})(\bar{P}_{24}\vee\bar{P}_{25})(P_{24}\vee P_{26})(\bar{P}_{25}\vee\bar{P}_{26})$

$\Leftrightarrow(P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee\bar{P}_{24}P_{25}P_{26})(\bar{P}_{24}\bar{P}_{25}P_{26}\vee P_{24}\bar{P}_{25}\vee\bar{P}_{25}P_{26}\vee P_{24}\bar{P}_{25}\bar{P}_{26})\Leftrightarrow P_{23}\bar{P}_{24}\bar{P}_{25}P_{26}$

$\Leftrightarrow(\bar{P}_{12}\vee\bar{P}_{13})(P_{12}\vee P_{14})(P_{12}\vee P_{15})(\bar{P}_{12}\vee\bar{P}_{16})\Leftrightarrow\bar{P}_{12}P_{14}P_{15}\vee P_{12}\bar{P}_{13}\bar{P}_{16}\vee\bar{P}_{13}P_{14}P_{15}\bar{P}_{16}$

$\Rightarrow(\bar{P}_{12}P_{14}P_{15}\vee P_{12}P_{13}\bar{P}_{16}\vee\bar{P}_{13}P_{14}P_{15}\bar{P}_{16})(\bar{P}_{13}\vee\bar{P}_{14})(P_{13}\vee P_{15})(P_{13}\vee P_{16})(\bar{P}_{14}\vee\bar{P}_{15})(P_{14}\vee P_{16})(\bar{P}_{15}\vee$
$\bar{P}_{16})\Leftrightarrow(\bar{P}_{12}P_{14}P_{16}\vee P_{12}\bar{P}_{13}\bar{P}_{16}\vee\bar{P}_{13}P_{14}P_{15}\bar{P}_{16})P_{13}\bar{P}_{14}\bar{P}_{15}P_{16}\Leftrightarrow F$，

$\bar{P}_{34}\bar{P}_{35}P_{36}\Rightarrow(P_{23}\vee P_{24})(P_{23}\vee P_{25})(\bar{P}_{23}\vee\bar{P}_{26})\Leftrightarrow P_{23}\bar{P}_{26}\vee\bar{P}_{23}P_{24}P_{25}\vee P_{24}P_{25}\bar{P}_{26}$

$\Leftrightarrow(P_{23}\bar{P}_{26}\vee\bar{P}_{23}P_{24}P_{25}\vee P_{24}P_{25}\bar{P}_{26})(\bar{P}_{24}\vee\bar{P}_{25})(P_{24}\vee P_{26})(\bar{P}_{25}\vee\bar{P}_{26})$

$\Leftrightarrow(P_{23}\bar{P}_{26}\vee\bar{P}_{23}P_{24}P_{25}\vee P_{24}P_{25}\bar{P}_{26})(\bar{P}_{24}\bar{P}_{25}P_{26}\vee P_{24}\bar{P}_{25}\vee\bar{P}_{25}P_{26}\vee P_{24}\bar{P}_{25}\bar{P}_{26})\Leftrightarrow P_{23}P_{24}\bar{P}_{25}\bar{P}_{26}$

$\Rightarrow(\bar{P}_{12}\vee\bar{P}_{13})(\bar{P}_{12}\vee\bar{P}_{14})(P_{12}\vee P_{15})(P_{12}\vee P_{16})(P_{13}\vee P_{14})(P_{13}\vee P_{15})(\bar{P}_{13}\vee\bar{P}_{16})(\bar{P}_{14}\vee\bar{P}_{15})(P_{14}\vee$
$P_{16})(\bar{P}_{15}\vee\bar{P}_{16})\Leftrightarrow(\bar{P}_{12}P_{15}P_{16}\vee P_{12}\bar{P}_{13}\bar{P}_{14}\vee\bar{P}_{13}\bar{P}_{14}P_{15}P_{15})P_{13}P_{14}\bar{P}_{15}\bar{P}_{16}\Leftrightarrow F$，

3) $P_{56}\Rightarrow\bar{P}_{45}P_{46}\vee P_{45}\bar{P}_{46}\vee\bar{P}_{45}\bar{P}_{46}$

$\bar{P}_{45}\bar{P}_{46}\Rightarrow(P_{34}\vee P_{35})(P_{34}\vee P_{36})\Rightarrow(P_{34}\vee P_{35}P_{36})(\bar{P}_{35}\vee\bar{P}_{36})\Leftrightarrow P_{34}\bar{P}_{35}\vee P_{34}\bar{P}_{36}\Leftrightarrow P_{34}\bar{P}_{35}P_{36}\vee P_{34}P_{35}\bar{P}_{36}\vee$
$P_{34}\bar{P}_{35}\bar{P}_{36}$

$P_{34}\bar{P}_{35}P_{36}\Rightarrow(\bar{P}_{23}\vee\bar{P}_{24})(P_{23}\vee P_{25})(\bar{P}_{23}\vee\bar{P}_{26})(P_{24}\vee P_{25})(P_{24}\vee P_{26})(\bar{P}_{25}\vee\bar{P}_{26})$

$\Leftrightarrow(\bar{P}_{23}P_{25}\vee P_{23}\bar{P}_{24}\bar{P}_{26}\vee\bar{P}_{24}P_{25}\bar{P}_{26})(P_{24}\bar{P}_{25}\vee P_{24}\bar{P}_{26})\Leftrightarrow\bar{P}_{23}P_{24}P_{25}\bar{P}_{26}$

$\Leftrightarrow(P_{12}\vee P_{13})(\bar{P}_{12}\vee\bar{P}_{14})(\bar{P}_{12}\vee\bar{P}_{15})(P_{12}\vee P_{16})\Leftrightarrow P_{12}\bar{P}_{14}\bar{P}_{15}\vee\bar{P}_{12}P_{13}P_{16}\vee P_{13}\bar{P}_{14}\bar{P}_{15}P_{16}$

$\Rightarrow(P_{12}\bar{P}_{14}\bar{P}_{15}\vee\bar{P}_{12}P_{13}P_{16}\vee P_{13}\bar{P}_{14}\bar{P}_{15}P_{16})(\bar{P}_{13}\vee\bar{P}_{14})(P_{13}\vee P_{15})(\bar{P}_{13}\vee\bar{P}_{16})(P_{14}\vee P_{15})(P_{14}\vee P_{15})(\bar{P}_{15}\vee$
$\bar{P}_{16})\Leftrightarrow(P_{12}\bar{P}_{14}\bar{P}_{15}\vee\bar{P}_{12}P_{13}P_{15}\vee P_{13}\bar{P}_{14}\bar{P}_{15}P_{16})(\bar{P}_{13}P_{15}\vee P_{13}\bar{P}_{14}\bar{P}_{16}\vee P_{14}P_{15}\bar{P}_{16})(P_{14}\bar{P}_{15}\vee P_{14}\bar{P}_{16})$

$\Leftrightarrow P_{12}P_{13}\bar{P}_{14}\bar{P}_{15}\bar{P}_{16}(P_{14}\bar{P}_{15}\vee P_{14}\bar{P}_{16})\Leftrightarrow F$，

$P_{34}P_{35}\bar{P}_{36}\Rightarrow(\bar{P}_{23}\vee\bar{P}_{24})(\bar{P}_{23}\vee\bar{P}_{25})(P_{23}\vee P_{26})\Rightarrow(\bar{P}_{23}P_{26}\vee P_{23}\bar{P}_{24}\bar{P}_{25}\vee\bar{P}_{24}\bar{P}_{25}P_{26})(P_{24}\vee P_{25})(P_{24}\vee P_{26})(\bar{P}_{25}$
$\vee\bar{P}_{26})\Leftrightarrow(\bar{P}_{23}P_{26}\vee P_{23}\bar{P}_{24}\bar{P}_{25}\vee\bar{P}_{24}\bar{P}_{25}P_{26})(P_{24}\bar{P}_{25}\vee P_{24}\bar{P}_{26})\Leftrightarrow\bar{P}_{23}P_{24}\bar{P}_{25}P_{26}$

$\Rightarrow(P_{12}\vee P_{13})(\bar{P}_{12}\vee\bar{P}_{14})(P_{12}\vee P_{15})(\bar{P}_{12}\vee\bar{P}_{16})\Leftrightarrow P_{12}\bar{P}_{14}\bar{P}_{16}\vee\bar{P}_{12}P_{13}P_{15}\vee P_{13}\bar{P}_{14}P_{15}\bar{P}_{16}$

$\Rightarrow(P_{12}\bar{P}_{14}\bar{P}_{16}\vee\bar{P}_{12}P_{13}P_{15}\vee P_{13}\bar{P}_{14}P_{15}\bar{P}_{16})(\bar{P}_{13}\vee\bar{P}_{14})(\bar{P}_{13}\vee\bar{P}_{15})(P_{13}\vee P_{16})(P_{14}\vee P_{15})(P_{14}\vee P_{16})(\bar{P}_{15}\vee$
$\bar{P}_{16})\Leftrightarrow(P_{12}\bar{P}_{14}\bar{P}_{16}\vee\bar{P}_{12}P_{13}P_{15}\vee P_{13}\bar{P}_{14}P_{15}\bar{P}_{16})(\bar{P}_{13}P_{16}\vee P_{13}\bar{P}_{14}\bar{P}_{15}\vee\bar{P}_{14}\bar{P}_{15}P_{16})(P_{14}\bar{P}_{15}\vee P_{14}\bar{P}_{16})$

$\Leftrightarrow P_{12}P_{13}\bar{P}_{14}\bar{P}_{15}\bar{P}_{16}(P_{14}\bar{P}_{15}\vee P_{14}\bar{P}_{16})\Leftrightarrow F$，

$P_{34}\bar{P}_{35}\bar{P}_{36}\Rightarrow(\bar{P}_{23}\vee\bar{P}_{24})(P_{23}\vee P_{25})(P_{23}\vee P_{26})\Leftrightarrow P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee\bar{P}_{24}P_{25}P_{26}$

$\Rightarrow(P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee\bar{P}_{24}P_{25}P_{26})(P_{24}\vee P_{25})(P_{24}\vee P_{26})(\bar{P}_{25}\vee\bar{P}_{26})$

$\Leftrightarrow(P_{23}\bar{P}_{24}\vee\bar{P}_{23}P_{25}P_{26}\vee\bar{P}_{24}P_{25}P_{26})(P_{24}\bar{P}_{25}\vee P_{24}\bar{P}_{26})\Leftrightarrow F$.

类似结果对于 $\overline{P}_{56}$ 的假设同样成立(略). 以上说明,任意 6 个顶点的图都不可能既不含 K_3,也不含 $\overline{K}_3$,即证明了 6 是 Ramsey 数: $R(K_3,\overline{K}_3)=6$.

一般,对于 $R(K_n,\overline{K}_m)$ 的计算,常用的是(4)、(5) 式,例如对 K_4 和 $\overline{K}_3$,我们有

	2	3	4	5	6	7	条件
1	1	1	0	0	1	0	
2		0	1	0	0	1	($i<2$)2,4,7 列(处)不全为 1;2,3;2,5 与 2,6 列不全为 0.
3			0	1	1	0	($i<3$)3,5,6 列(处)不全为 1;3 和 4 与 3 和 7 列(处)不全为 0.
4				1	0	1	($i<4$)4,5,7 列(处)不全为 1;4,6 列(处)不全为 0.
5					1	1	($i<5$)5,6,7 列不全为 1.(其中 i 表示行序).
6						1	$\therefore R(K_4,\overline{K}_3)\neq 7$.

	2	3	4	5	6	7	8	条件
1	1	1	1	1	0	0	0	
2		1	1	0	0	1	0	($i<2$)2、4、7 列不全为 1;2,5;2,6 与 2,8 列不全为 0.
3			0	1	0	0	1	($i<3$)3,5,8 列不全为 1;3,4;3,6 与 3,7 列不全为 0.
4				0	1	1	0	($i<4$)4,6,7 列不全为 1;3,5 与 3,8 列不全为 0.
5					1	0	1	($i<5$)5,6,8 列不全为 1;5,7 列不全为 0.
6						1	1	($i<6$)6,7,8 列不全为 1.(其中 i 为行序).
							1	$\therefore R(K_4,\overline{K}_3)\neq 8$.

对于 9,这样的矩阵已不存在,说明有可能 $R(K_4,\overline{K}_3)=9$. 此时,需用命题演算法严格证明之(略). 具体操作我们将另文探索.

由此看出,用此方法求 Ramsey 数时,前一矩阵可作后面矩阵之子矩阵而被利用. 其简捷性亦在乎此.

显然,以上方法适用于求任意一个 Ramsey 数. 所不同的仅在于(4)式和(5)式中的 n 和 m 而已.

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