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时滞扩散的竞争系统的持久性和全局吸引性

Persistence and Attractiveness of A Delay Competitive System with Diffusion

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摘要　研究缀块环境下两种群时滞扩散的 Lotka-Volterra竞争系统,获得了系统的一致持久性和正解的全局吸引性的条件.

关键词　持久性　竞争系统　全局吸引

中图法分类号　O 175.21

Abstract　A delay Lotka-Volterra competitive system with diffusion in a patch environment is considered. Uniform persistence and global attractiveness of this system are obtained.

Key words　persistence, competitive system, global attractiveness

自从 20多年前 Levin[1]建立了缀块环境生态系统的 Lotka-Volterra模型以来，关于这模型所描述的种群动力学行为的研究越来越引起中外数学工作者的兴趣[2,3],不久前 G. Zeng和 L. Chen[4]研究了两种群竞争扩散的 Lotka-Volterra系统的持久性的周期性态，在本文中，我们进一步假定文献 [4] 中的系统和过去的状态有关. 即我们考虑下列系统

$$\begin{cases} \dot{x}_1 = x_1 [r_1(t) - a_1(t)x_1 - b_1(t)y - \mathrm{T}(t)\int_{-r}^{0} k_1(s)x_1(t+s)\,\mathrm{d}s] + D_1(t)(x_2 - x_1) \\ \dot{x}_2 = x_2 [r_2(t) - a_2(t)x_2 - \mathrm{U}(t)\int_{-r}^{0} k_2(s)x_2(t+s)\,\mathrm{d}s] + D_2(t)(x_1 - x_2) \\ \dot{y} = y [r_3(t) - a_3(t)x_1 - b_3(t)y] \end{cases} \tag{1}$$

对于系统(1),我们作如下假设:

(H_1)　$r_i(t), a_i(t)(i=1,2,3), b_i(t)(i=1,3), D_i(t)(i=1,2)$及 $\mathrm{T}(t), \mathrm{U}(t)$都是 $t\in[0,\infty)$的连续函数,且都有正的上下界.

(H_2)　$k_i(s)\geqslant 0$,当 $s\in[-r,0]$时,其中 $r\geqslant 0$, $k_i(s)$分段连续且 $\int_{-r}^{0} k_i(s)\,\mathrm{d}s = 1$.

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设 $R_+^3 = \{x = (x_1, x_2, y) | x_i \geqslant 0 \ (i = 1,2), y \geqslant 0\}$.如果 $x \in \mathrm{Int}\, R_+^3 = \{x | x_i > 0 \ (i = 1,2), y > 0\}$,则记为 $x > 0$. $C([-r,0], R_+^3)$表示 $[-r,0]$上的非负连续函数所成的 Banach空间,其中的范数为 $\|\varphi\| = \sup_{s \in [-r,0]} |\varphi(s)|$.在本文中,取 $C^+ = \{\varphi \in C([-r,0], R_+^3) \,|\, \varphi(0) > 0\}$作为(1)的状态空间,易于证明,$\forall \varphi \in C^+$,$\exists A \in (0, +\infty)$及唯一的定义于 $[-r, A]$上的(1)解 $x(t,\varphi)$,满足 $x(t,\varphi) > 0, t \in [0, A]$,这样的解称为系统(1)的正解.考虑到生态意义,我们只研究(1)的正解的性质.此外,对有界函数 $f(t)$,记

$$f^M = \sup_{t \in [0,+\infty)} f(t), f^L = \inf_{t \in [0,+\infty)} f(t).$$

1　一致持久性

引理 1　设 $\varphi \in C^+$,$x(t,\varphi) = (x_1(t), x_2(t), y(t))$是(1)的正解,则存在 $T \geqslant 0$,使得当 $t \geqslant T$时,

$$x_i(t) \leqslant M_2 (i = 1,2), y(t) \leqslant y_2,$$

其中 $M_2 > M_2^* = \max\{r_1^M / a_1^L, r_2^M / a_2^L\}, y_2 > y_2^* = r_3^M / b_3^L$.

证明　由(1)知 $\dot{y} \leqslant y[r_3(t) - b_3(t) y] \leqslant y(r_3^M - b_3^L)$,

于是若 $y(t) \geqslant y_2$,则 $\dot{y} \leqslant y_2 (r_3^M - b_3^L y_2) < 0$,所以存在 $T_1 \geqslant 0$,使当 $t \geqslant T_1$时,$y(t) \leqslant y_2$.

令 $V(t) = \max\{x_1(t), x_2(t)\}$,则

$$D^+ V(t) \leqslant V(t)[r_1^M - a_1^L V(t)],\text{当 } x_1(t) \geqslant x_2(t)\text{ 时},$$

或 $D^+ V(t) \leqslant V(t)[r_2^M - a_2^L V(t)]$,当 $x_1(t) < x_2(t)$时.

取 $\lambda = \max\{M_2(r_1^M - a_1^L M_2), M_2(r_2^M - a_2^L M_2)\}$,于是若 $V(t) \geqslant M_2$,则 $D^+ V(t) \leqslant \lambda < 0$.所以存在 $T_2 \geqslant 0$,当 $t \geqslant T_2$时,$V(t) \leqslant M_2$.取 $T = \max\{T_1, T_2\}$,则当 $t \geqslant T$时,$x_i(t) \leqslant M_2 (i = 1, 2), y(t) \leqslant y_2$.证毕.

定理 1　设系统(1)满足如下条件

(H_3)　$r_3^L / \mathrm{T}_3^M > M_2^*$;

(H_4)　$r_2^L / \mathrm{U}^M > M_2^*$, $r_1^L / (b_1^M + \mathrm{T}^M) > \max\{y_2^*, M_2^*\}$.

其中 y_2^*, M_2^* 见引理 1.则系统(1)一致持久.

证明　由(1)及引理 1知,当 $t \geqslant T$时,

$$\dot{y} \geqslant y(r_3^L - a_3^M M_2 - b_3^M y), \tag{2}$$

由引理 1的证明可以看出,我们可以选择 M_2使之充分接近 M_2^*,例如可使之满足 $M_2 \in (M_2^*, r_3^L / \mathrm{T}_3^M)$,于是 $r_3^L - \mathrm{T}_3^M M_2 > 0$.取 $y_1 \in (0, (r_3^L - \mathrm{T}_3^M M_2) / b_3^M)$,注意当 $0 < y(t) < y_1$时,$\dot{y} > 0$,因而 $y(0) < y(t)$.所以若是 $0 < y(t) < y_1$,则

$$\dot{y} \geqslant y(t)[r_3^L - \mathrm{T}_3^M M_2 - b_3^M y_1] > y(0)[r_3^L - \mathrm{T}_3^M M_2 - b_3^M y_1] > 0,$$

从而 $\exists T_3 \geqslant T$,当 $t \geqslant T_3$时,$y(t) \geqslant y_1$.

令 $V_1(t) = \min\{x_1(t), x_2(t)\}$,计算并估计 $V_1(t)$的下右导数 $D_+ V_1(t)$,当 $t \geqslant T + r$时有

$$D_+ V_1(t) = \dot{x}_1(t) \geqslant x_1[r_1(t) - a_1(t)x_1 - b_1(t)y - \mathrm{T}(t)\int_{-r}^{0} k_1(s) x_1(t+s)\,ds] \geqslant x_1(r_1^L - b_1^M y_2$$

$$- \mathrm{T}^M M_2 - a_1^M x_1) \geqslant V_1(t)[r_1^L - b_1^M y_2 - \mathrm{T}^M M_2 - a_1^M V_1(t)],\quad \text{当 } x_1(t) \leqslant x_2(t)\text{ 时} \tag{3}$$

或

从引理 1的证明过程知,(3)(4)中的 y_2, M_2可以选取得使它们分别充分接近 y_2^* 和 M_2^*,例如可取 $y_2 \in (y_2^*, r_1^L/(b_1^M + T^M))$ 而 $M_2 \in (M_2^*, \min\{r_1^L/(b_1^M + T^M), r_2^L/U^M\})$,于是有 $r_1^L - b_1^M y_2 - T^M M_2 > 0$和 $r_2^L - U^M M_2 > 0$.取 M_1满足

$$0 < M_1 < \min\{(r_1^L - b_1^M y_2 - T^M M_2)/T_1^M, (r_2^L - U^M M_2)/T_2^M\}$$

令 $\_ = \min\{V_1(0)(r_2^L - U^M M_2 - T_2^M M_1), V_1(0)(r_1^L - b_1^M y_2 - T^M M_2 - T_1^M M_1)\}$,

则若 $0 < V_1(t) < M_1$,就有 $D_+ V_1(t) \geqslant \_ > 0$.于是存在 $T_4 \geqslant T + r$,当 $t \geqslant T_4$时,$V_1(t) \geqslant M_1$.

取 $T^* = \max[T_3, T_4]$,

令 $K = \{x = (x_1, x_2, y) | M_1 \leqslant x_i \leqslant M_2 (i = 1,2), y_1 \leqslant y \leqslant y_2\}$.

则当 $t \geqslant T^*$ 时,方程(1)的满足 $h \in C^+$ 的每一正解 $x(t, h) \in K$,故系统(1)一致持久.证毕.

2 全局吸引性

定义 (1)的解 $u(t) = (u_1(t), u_2(t), v(t))$ 是全局吸引的,如果对于任一个(1)的解 $x(t, h) = (x_1(t), x_2(t), y(t))$, $h \in C^+$,都有

$$\lim_{t \to +\infty} |u_i(t) - x_i(t)| = 0 \quad (i = 1, 2), \lim_{t \to +\infty} |v(t) - y(t)| = 0$$

引理 2[5] 若定义于 $[0, +\infty)$ 的非负可积函数 $f(t)$ 于 $[0, +\infty)$ 一致连续,则 $\lim_{t \to +\infty} f(t) = 0$.

定理 2 若系统(1)满足(H_1)~(H_4)和

(H_5) $a_1^L > a_3^M + a^M + D_2^M/M_1$, $a_2^L > U^M + D_1^M/M_1$, b_3^L / b_1^M,

则系统(1)的每个正解对于位于 R_+^3 的其他解而言是全局吸引的.

证明 设 $x(t, h_1) = (x_1(t), x_2(t), y(t))$和 $u(t, h_2) = (y_1(t), y_2(t), v(t))$是(1)的解,$h_1, h_2 \in C^+$.根据定理 1,存在 T^*,当 $t \geqslant T^*$ 时,$x(t) = x(t, h_1)$和 $u(t) = u(t, h_2)$都在 K中.

作 Lyapunov函数

$$V_2(t) = |\ln x_1(t) - \ln u_1(t)| + |\ln x_2(t) - \ln u_2(t)| + |\ln y(t) - \ln v(t)| + T^M \int_{-r}^{0} k_1(s) \int_{t+s}^{t} |x_1(\theta) - u_1(\theta)| d\theta ds + U^M \int_{-r}^{0} k_2(s) \int_{t+s}^{t} |x_2(\theta) - u_2(\theta)| d\theta ds.$$

因为

$$D^+ |\ln x_1(t) - \ln u_1(t)| = [\dot{x}_1(t)/x_1(t) - \dot{u}_1(t)/u_1(t)] e [x_1(t) - u_1(t)] = \{-a_1(t)[x_1(t) - u_1(t)] - b_1(t)[y(t) - v(t)] - T(t)\int_{-r}^{0} k_1(s)[x_1(t+s) - u_1(t+s)] ds + D_1(t)[x_2(t)/x_1(t) - u_2(t)/u_1(t)]\} e [x_1(t) - u_1(t)] \leqslant - a_1^L |x_1(t) - u_1(t)| + b_1^M |y(t) - v(t)| + T^M \int_{-r}^{0} k_1(s) |x_1(t+s) - u_1(t+s)| ds + \tilde{D}_1(t),$$

其中 e见文献[6],而

$$\tilde{D}_1(t) = D_1(t)[x_2(t)/x_1(t) - u_2(t)/u_1(t)] e [x_1(t) - u_1(t)],$$

$$D^+ |\ln x_2(t) - \ln u_2(t)| = [\dot{x}_2(t)/x_2(t) - \dot{u}_2(t)/u_2(t)] e [x_2(t) - u_2(t)] \leqslant - a_2^L |x_2(t) - u_2(t)| + U^M \int_{-r}^{0} k_2(s) |x_2(t+s) - u_2(t+s)| ds + \tilde{D}_2(t),$$

… .

$$\tilde{D}_2(t) = D_2(t)[x_1(t)/x_2(t) - u_1(t)/u_2(t)]^{e}[x_2(t) - u_2(t)],$$

以及

$$D^+|\ln y(t) - \ln v(t)| \leqslant a_3^M|x_1(t) - u_1(t)| + b_3^L|y(t) - v(t)|,$$

又由 ^{e}函数的定义,我们有

$$\tilde{D}_1(t) \leqslant (D_1^M/M_1)|x_2(t) - u_2(t)|, \tilde{D}_2(t) \leqslant (D_2^M/M_1)|x_1(t) - u_1(t)|,$$

当 $t \geqslant T^*$ 时成立.所以对 $V_2(t)$求上右导数得

$$D^+ V_2(t) \leqslant -(a_1^L - a_3^M - T^M - D_2^M/M_1)|(x_1(t) - u_1(t)| - (a_2^L - U^M - D_1^M/M_1)|(x_2(t) - u_2(t)| - (b_3^L - b_1^M)|y(t) - v(t)| \leqslant -\_(|x_1(t) - u_1(t)| + |x_2(t) - u_2(t)| + |y(t) - v(t)|), \quad t \geqslant T,$$

其中 $\_ = \min\{a_1^L - a_3^M - T^M - D_2^M/M_1, a_2^L - U^M - D_1^M/M_1, b_3^L - b_1^M\} > 0.$

由微分不等式,

$$V_2(t) + \_\int_{T^*}^{t}(|x_1(s) - u_1(s)| + |x_2(s) - u_2(s)| + |y(s) - v(s)|)\mathrm{d}s \leqslant V_2(T^*),$$

于是

$$\int_{T^*}^{t}(|x_1(s) - u_1(s)| + |x_2(s) - u_2(s)| + |y(s) - v(s)|)\mathrm{d}s \leqslant V_2(T^*)/\_,$$

即

$$\int_{T^*}^{+\infty}(|x_1(s) - u_1(s)| + |x_2(s) - u_2(s)| + |y(s) - v(s)|)\mathrm{d}s < +\infty.$$

因为 $x(t)$和 $u(t)$对于 $t \geqslant T^*$ 是有界的,由(1)知$\frac{\mathrm{d}}{\mathrm{d}t}(x_i(t) - u_i(t))(i = 1,2)$和$\frac{\mathrm{d}}{\mathrm{d}t}(y(t) - v(t))$在 $t \geqslant T^*$ 时也有界.因而

$$|x_1(t) - u_1(t)| + |x_2(t) - u_2(t)| + |y(t) - v(t)|$$

在 $[T^*, \infty)$一致连续,由引理 2,有

$$\lim_{t \to +\infty}(|x_1(t) - u_1(t)| + |x_2(t) - u_2(t)| + |y(t) - v(t)|) = 0.$$

证毕.

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