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Erlang(2)风险模型的 Gerber-Shiu 折扣罚函数满足的更新方程*

The Renewal Function of Erlang(2) Risk Model with Satisfied Gerber-Shiu Discounted Penalty Function

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摘要:通过 Gerber-Shiu 折扣罚函数对索赔量与索赔时间相依的 Erlang(2)风险模型进行分析,并利用 Dickson-Hipp 算子得到 Gerber-Shiu 折扣罚函数满足的更新方程.

关键词:Gerber-Shiu 折扣罚函数 Dickson-Hipp 算子 相依 Erlang(2)过程
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Abstract: The Gerber-Shiu discounted penalty function is investigated in the risk model with Erlang(2) process for inter-claim-dependent claim sizes. The renewal function that satisfies the Gerber-Shiu discounted penalty function is obtained.

Key words: Gerber-Shiu discounted penalty function, Diskon-Hipp operator, inter-dependent, Erlang(2) process

近年已经有很多学者对风险模型进行深入的研究,得到了一些很好的结果. 例如,Cheng 等[1]得到风险模型下破产概率的明确表达式,以及破产时刻的一阶矩. Boudreault[2]得到风险模型破产前的余额及破产前赤字的联合分布. 事实上,在很多情况下,索赔量与索赔时间之间并不是相互独立的. Dickson 等[3]已经注意到这个问题,也对该问题进行研究,得到了当索赔量与索赔时间满足特殊相依关系时,Gerber-Shiu 折扣罚函数满足的更新方程. 本文进一步对索赔量与索赔时间相依的 Erlang(2)风险模型进行分析,得到了 Gerber-Shiu 折扣罚函数满足的更新方程.

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1 基本引理

设 t 时刻的余额过程[3]可以表示为

$$U(t)=u+ct-\sum_{i=1}^{N(t)}X_i, \qquad (1)$$

那么索赔量与索赔时间相依是指:二维随机向量 (W_i, X_i) 对于不同的 $i\in N^+$ 是相互独立的,但是 W_i 与 X_i 是相关的. 文献[3]构造了一种特殊的 W_i 和 X_i 之间的相依关系,即假设当 $W_i=t$ 时,随机变量的条件密度函数 $f_{X_i|W_i(x)}$ 按下面的式子定义:

$$f_{X_i|W_i(x)}=e^{-\beta t}f_1(x)+(1-e^{-\beta t})f_2(x), x\geqslant 0. \qquad (2)$$

而且是在 Erlang(2)风险模型中索赔量与索赔时间满足(2)式时进行研究. 另外定义几个变量:$T=\inf\{t: U(t)<0\}$ 表示破产时间;设 $\omega(x_1,x_2)$,$0\leqslant x_1,x_2<\infty$ 是非负函数(即破产罚函数),对于 $\delta\geqslant 0$,定义

$$m_\delta(u)=E[e^{-\delta T}\omega(U(T-),|U(T)|)I(T<\infty)|U(0)=u]$$

为折扣罚函数,其中 δ 是利息力度,$I(x)$ 是示性

函数.

推导折扣罚函数 $m_\delta(u)$ 满足的积分方程. 当 $u \geqslant 0$ 时,根据首次索赔到达时间 t,以及首次索赔额 y,分情况讨论可得

$$m_\delta(u) = \int_0^{+\infty} e^{-\delta t}k(t)\int_0^{u+ct} m_\delta(u+ct-y) \times (e^{-\beta t}f_1(y)+(1-e^{-\beta t})f_2(y))\mathrm{d}y\mathrm{d}t + \int_0^{+\infty} e^{-\delta t}k(t)\int_{u+ct}^{\infty}\omega(u+ct, y-(u+ct)) \times (e^{-\beta t}f_1(y)+(1-e^{-\beta t})f_2(y))\mathrm{d}y\mathrm{d}t = \int_0^{+\infty}\lambda^2 te^{-(\lambda+\delta)t}k(t)\int_0^{u+ct} m_\delta(u+ct-y)\times (e^{-\beta t}f_1(y)+(1-e^{-\beta t})f_2(y))\mathrm{d}y\mathrm{d}t + \int_0^{+\infty}\lambda^2 te^{-(\lambda+\delta)t}k(t)\int_{u+ct}^{\infty}\omega(u+ct, y-(u+ct))\times (e^{-\beta t}f_1(y)+(1-e^{-\beta t})f_2(y))\mathrm{d}y\mathrm{d}t = \frac{\lambda^2}{c}\int_u^{+\infty}(\frac{t-u}{c})e^{-\frac{\lambda+\delta+\beta}{c}(t-u)}\times(\sigma_{1,\delta}(t)-\sigma_{2,\delta}(t))\mathrm{d}t + \frac{\lambda^2}{c}\int_u^{+\infty}(\frac{t-u}{c})e^{-\frac{\lambda+\delta}{c}(t-u)}\times\sigma_{2,\delta}(t)\mathrm{d}t, \quad (3)$$

其中,$u \geqslant 0$,

$$\sigma_{i,\delta}(u) = \int_0^u m_\delta(u-y)f_i(y)\mathrm{d}y + \gamma_i(u), \quad (4)$$

$$\gamma_i(u) = \int_u^{\infty}\omega(y, y-u)f_i(y)\mathrm{d}y, i=1,2.$$

对(3)式两边分别求导可得

$$m_\delta'(u) = -\frac{1}{c}\frac{\lambda^2}{c}\int_u^{\infty}e^{-\frac{\lambda+\delta+\beta}{c}(t-u)}\times(\sigma_{1,\delta}(t)-\sigma_{2,\delta}(t))\mathrm{d}t + \frac{\lambda+\delta+\beta}{c}\frac{\lambda^2}{c}\int_u^{\infty}\frac{t-u}{c}e^{-\frac{\lambda+\delta+\beta}{c}(t-u)}\times(\sigma_{1,\delta}(t)-\sigma_{2,\delta}(t))\mathrm{d}t - \frac{1}{c}\frac{\lambda^2}{c}\int_u^{\infty}e^{-\frac{\lambda+\delta}{c}(t-u)}\times\sigma_{2,\delta}(t)\mathrm{d}t + \frac{\lambda+\delta}{c}\frac{\lambda^2}{c}\int_u^{\infty}\frac{t-u}{c}e^{-\frac{\lambda+\delta}{c}(t-u)}\times\sigma_{2,\delta}(t)\mathrm{d}t = \frac{\lambda+\delta}{c}m_\delta(u) - \frac{1}{c}\frac{\lambda^2}{c}\int_u^{\infty}e^{-\frac{\lambda+\delta+\beta}{c}(t-u)}\times(\sigma_{1,\delta}(t)-\sigma_{2,\delta}(t))\mathrm{d}t - \frac{1}{c}\frac{\lambda^2}{c}\int_u^{\infty}e^{-\frac{\lambda+\delta}{c}(t-u)}\times\sigma_{2,\delta}(t)\mathrm{d}t - \frac{\beta}{c}\frac{\lambda^2}{c}\int_u^{\infty}(\frac{t-u}{c})e^{-\frac{\lambda+\delta}{c}(t-u)}\times\sigma_{2,\delta}(t)\mathrm{d}t.$$

同理可得

$$m_\delta''(u) = \frac{2\lambda+2\delta+\beta}{c}m_\delta'(u) - \frac{\lambda+\delta}{c}\frac{\lambda+\delta+\beta}{c}m_\delta(u) + \frac{1}{c}\frac{\lambda^2}{c}\sigma_{1,\delta}(t) - \frac{1}{c}\frac{\lambda^2}{c}\frac{\beta}{c}\int_u^{\infty}e^{-\frac{\lambda+\delta+\beta}{c}(t-u)}\times(\sigma_{1,\delta}(t)-\sigma_{2,\delta}(t))\mathrm{d}t + \frac{\beta}{c}\frac{\lambda^2}{c}\frac{\beta}{c}\int_u^{\infty}(\frac{t-u}{c})e^{-\frac{\lambda+\delta}{c}(t-u)}\times\sigma_{2,\delta}(t)\mathrm{d}t. \quad (5)$$

运用 Dickson-Hipp 算子 T_r,其中 T_r 定义为

$$T_r f(u) = \int_u^{\infty}e^{-r(y-u)}f(y)\mathrm{d}y.$$

则(5)式变为

$$m_\delta''(u) = \frac{2\lambda+2\delta+\beta}{c}m_\delta'(u) - \frac{\lambda+\delta}{c}\frac{\lambda+\delta+\beta}{c}m_\delta(u) + \frac{1}{c}\frac{\lambda^2}{c}\sigma_{1,\delta}(u) - \frac{1}{c}\frac{\lambda^2}{c}\frac{\beta}{c}(T_{\frac{\lambda+\delta+\beta}{c}}\sigma_{1,\delta}(u) - T_{\frac{\lambda+\delta+\beta}{c}}\sigma_{2,\delta}(u) - T_{\frac{\lambda+\delta}{c}}\sigma_{1,\delta}(u)).$$

对上式两边作 Laplace 变换,并且运用文献[4]中性质 2 可得

$$s^2\tilde{m}_\delta(s) - sm_\delta(0) - m_\delta'(0) = \frac{2\lambda+2\delta+\beta}{c}\cdot(s\tilde{m}_\delta(s)-m_\delta(0)) - \frac{\lambda+\delta}{c}\frac{\lambda+\delta+\beta}{c}\tilde{m}_\delta(s) + \frac{1}{c}\frac{\lambda^2}{c}\tilde{\sigma}_{1,\delta}(s) - \frac{1}{c}\frac{\lambda^2}{c}\frac{\beta}{c}\left[\frac{\tilde{\sigma}_{1,\delta}(\frac{\lambda+\delta+\beta}{c})-\tilde{\sigma}_{1,\delta}(s)}{s-\frac{\lambda+\delta+\beta}{c}} - \frac{\tilde{\sigma}_{2,\delta}(\frac{\lambda+\delta+\beta}{c})-\tilde{\sigma}_{2,\delta}(s)}{s-\frac{\lambda+\delta+\beta}{c}} - \frac{\tilde{\sigma}_{2,\delta}(\frac{\lambda+\delta}{c})-\tilde{\sigma}_{2,\delta}(s)}{s-\frac{\lambda+\delta}{c}}\right].$$

整理得

$$\tilde{m}_\delta(s) = [\frac{1}{c}\frac{\lambda^2}{c}(s-\frac{\lambda+\delta}{c})^2\tilde{\sigma}_{1,\delta}(s) - 2\frac{1}{c}\frac{\lambda^2}{c}\frac{\beta}{c}(s-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2\tilde{\sigma}_{2,\delta}(s) + \bar{\alpha}_\delta(s)]/(s-\frac{\lambda+\delta}{c})^2(s-\frac{\lambda+\delta+\beta}{c})^2,$$

其中

$$\bar{\alpha}_\delta(s) = (s-\frac{\lambda+\delta}{c})(s-\frac{\lambda+\delta+\beta}{c})(s-\frac{2\lambda+2\delta+\beta}{c})m_\delta(0) + (s-\frac{\lambda+\delta}{c})(s-\frac{\lambda+\delta+\beta}{c})m_\delta'(0) - \frac{\beta}{c}\frac{\lambda^2}{c}(s-\frac{\lambda+\delta}{c})\tilde{\sigma}_{1,\delta}(\frac{\lambda+\delta+\beta}{c}) + (s-\frac{\lambda+\delta}{c})\tilde{\sigma}_{2,\delta}(\frac{\lambda+\delta+\beta}{c}) + (s-\frac{\lambda+\delta+\beta}{c})\cdot\tilde{\sigma}_{2,\delta}(\frac{\lambda+\delta}{c}). \quad (6)$$

而由(4)式知,$\tilde{\sigma}_{1,\delta}(s) = \tilde{m}_\delta(s)\tilde{f}_i(s) + \tilde{\gamma}_i(s)$. 那么

$$\tilde{m}_\delta(s) = \frac{\tilde{\beta}_\delta(s)+\tilde{\alpha}_\delta(s)}{\tilde{h}_{1,\delta}(s)-\tilde{h}_{2,\delta}(s)},$$

其中

$$\tilde{\beta}_\delta(s) = \frac{1}{c}\frac{\lambda^2}{c}(s-\frac{\lambda+\delta}{c})^2\tilde{\gamma}_1(s) -$$

$$2\frac{1}{c}\frac{\lambda^2}{c}\frac{\beta}{c}(s-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2\tilde{\gamma}_2(s);$$

$$\tilde{h}_{1,\delta}(s)=\left(s-\frac{\lambda+\delta}{c}\right)^2(s-\frac{\lambda+\delta+\beta}{c})^2;$$

$$\tilde{h}_{2,\delta}(s)=\frac{1}{c}\frac{\lambda^2}{c}(s-\frac{\lambda+\delta}{c})^2\tilde{f}_1(s)-$$

$$2\frac{1}{c}\frac{\lambda^2}{c}\frac{\beta}{c}(s-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2\tilde{f}_2(s). \quad (7)$$

引理 1[3] 若 $\delta>0$，则 $\tilde{h}_{1,\delta}(s)-\tilde{h}_{2,\delta}(s)=0$ 在有界区域 $D_\delta=\{s:Z_\delta(s)<1\}$ 内存在 4 个复根，分别记为 $s_1=s_1(\delta),s_2=s_2(\delta),s_3=s_3(\delta),s_4=s_4(\delta)$，其中 $Z_\delta(s)=\frac{\lambda+\delta+\beta-cs}{\lambda+\beta}$.

引理 2[5] 若 $\delta=0$，则 $\tilde{h}_{(1,0)}(s)-\tilde{h}_{(2,0)}(s)=0$ 在有界区域 $D_0=\{s:Z_0(s)<1\}$ 内存在 3 个复根，分别记为 $s_1=s_1(0),s_2=s_2(0),s_3=s_3(0)$，其中 $Z_0(s)=\frac{\lambda+\beta-cs}{\lambda+\beta}$，并且还有一个根为 $s_4=s_4(0)=0$.

引理 3[5] 设 s_1,s_2,s_3,s_4 为 4 个不同的非零实根，则 $m_\delta(s)$ 的拉普拉斯变换 $\tilde{m}_\delta(s)$ 满足方程：

$$\tilde{m}_\delta(s)=\tilde{m}_\delta(s)T_sT_{s_4}T_{s_3}T_{s_2}T_{s_1}h_{2,\delta}(0)+T_sT_{s_4}T_{s_3}T_{s_2}T_{s_1}\beta_\delta(0).$$

其中 T_r 为前面定义的 Dickson-Hipp 算子.

2 主要结论

定义

$$K_\delta=\frac{1}{c}\frac{\lambda^2}{c}((s_4-\frac{\lambda+\delta}{c})^2T_0T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_1(0)-2\frac{\beta}{c}(s_4-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2T_0T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_2(0)+2\frac{\beta}{c}T_0T_{s_3}T_{s_2}T_{s_1}f_2(0)-T_0T_{s_3}T_{s_2}T_{s_1}f_1(0);$$

$$q_{1,\delta}=\frac{\frac{1}{c}\frac{\lambda^2}{c}(s_4-\frac{\lambda+\delta}{c})^2T_0T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_1(0)}{K_\delta};$$

$$q_{2,\delta}=-2\cdot\frac{\frac{1}{c}\frac{\lambda^2}{c}\frac{\beta}{c}(s_4-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2T_0T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_2(0)}{K_\delta};$$

$$q_{3,\delta}=2\frac{\frac{1}{c}\frac{\lambda^2}{c}\frac{\beta}{c}T_0T_{s_3}T_{s_2}T_{s_1}f_2(0)}{K_\delta};$$

$$g_\delta(y)=q_{1,\delta}\frac{T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_1(y)}{T_0T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_1(0)}+q_{2,\delta}\frac{T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_2(y)}{T_0T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_2(0)}+q_{3,\delta}\frac{T_{s_3}T_{s_2}T_{s_1}f_2(y)}{T_0T_{s_3}T_{s_2}T_{s_1}f_2(0)}+(1-q_{1,\delta}-q_{2,\delta}-q_{3,\delta})\frac{T_{s_3}T_{s_2}T_{s_1}f_1(u)}{T_0T_{s_3}T_{s_2}T_{s_1}f_1(0)};$$

$$\xi_\delta(u)=\frac{1}{c}\frac{\lambda^2}{c}((s_4-\frac{\lambda+\delta}{c})^2T_{s_4}T_{s_3}T_{s_2}T_{s_1}\gamma_1(u)-2\frac{\beta}{c}(s_4-\frac{\lambda+\delta+\frac{\beta}{2}}{c})T_{s_4}T_{s_3}T_{s_2}T_{s_1}\gamma_2(u)+2\frac{\beta}{c}T_{s_3}T_{s_2}T_{s_1}\gamma_2(u)-T_{s_3}T_{s_2}T_{s_1}\gamma_1(u).$$

定理 设 s_1,s_2,s_3,s_4 为 4 个不同的非零实根，则折扣罚函数 $m_\delta(s)$ 满足下面的更新方程：

$$m_\delta(s)=K_\delta\int_0^u m_\delta(u-y)g_\delta(y)\mathrm{d}y+\xi_\delta(u).$$

其中 $K_\delta,g_\delta(y),\xi_\delta(u)$ 由前面的定义确定.

证明 由(7)式和文献[4]可得

$$T_sT_{s_4}T_{s_3}T_{s_2}T_{s_1}h_{2,\delta}(0)=\frac{\tilde{h}_{2,\delta}(s)}{\tau_4(s)}-\sum_{i=1}^{4}\frac{\tilde{h}_{2,\delta}(s_i)}{(s-s_i)\tau_4'(s_i)}=\frac{\frac{1}{c}\frac{\lambda^2}{c}(s-\frac{\lambda+\delta}{c})^2\tilde{f}_1(s)}{\tau_4(s)}-\sum_{i=1}^{4}\frac{\frac{1}{c}\frac{\lambda^2}{c}(s_i-\frac{\lambda+\delta}{c})^2\tilde{f}_1(s_i)}{(s-s_i)\tau_4'(s_i)}-$$

$$2\frac{\frac{\beta}{c}\frac{1}{c}\frac{\lambda^2}{c}(s-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2\tilde{f}_2(s)}{\tau_4(s)}+\sum_{i=1}^{4}\frac{\frac{\beta}{c}\frac{1}{c}\frac{\lambda^2}{c}(s_i-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2\tilde{f}_2(s_i)}{(s-s_i)\tau_4'(s_i)}=\frac{1}{c}\frac{\lambda^2}{c}(s_4-\frac{\lambda+\delta}{c})^2(\frac{\tilde{f}_1(s)}{\tau_4(s)}-\sum_{i=1}^{4}\frac{\tilde{f}_1(s_i)}{(s-s_i)\tau_4'(s_i)}-2\frac{\beta}{c}\frac{1}{c}\frac{\lambda^2}{c}(s_4-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2(\frac{\tilde{f}_2(s)}{\tau_4(s)}+\sum_{i=1}^{4}\frac{\tilde{f}_2(s_i)}{(s-s_i)\tau_4'(s_i)}-$$

$$2\frac{\beta}{c}\frac{1}{c}\frac{\lambda^2}{c}(\frac{\tilde{f}_2(s)}{\tau_4(s)}+\sum_{i=1}^{3}\frac{\tilde{f}_2(s_i)}{(s-s_i)\tau_3'(s_i)}-2\frac{1}{c}\frac{\lambda^2}{c}(\frac{\tilde{f}_1(s)}{\tau_4(s)}+\sum_{i=1}^{3}\frac{\tilde{f}_1(s_i)}{(s-s_i)\tau_3'(s_i)}=$$

$$\frac{1}{c}\frac{\lambda^2}{c}((s_4-\frac{\lambda+\delta}{c})^2T_0T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_1(0)-2\frac{\beta}{c}(s_4-\frac{\lambda+\delta+\frac{\beta}{2}}{c})^2T_0T_{s_4}T_{s_3}T_{s_2}T_{s_1}f_2(0)+2\frac{\beta}{c}T_0T_{s_3}T_{s_2}T_{s_1}f_2(0)-T_0T_{s_3}T_{s_2}T_{s_1}f_1(0). \quad (8)$$

其中 $\tau_j(s)=\sum_{i=1}^{j}(s-s_i)$.

同理可得

$$T_s T_{s_4} T_{s_3} T_{s_2} T_{s_1} \beta_\delta(0) = \frac{1}{c}\frac{\lambda^2}{c}((s_4 - \frac{\lambda+\delta}{c})^2 T_0 T_{s_4} T_{s_3} T_{s_2} T_{s_1} \gamma_1(0) - 2\frac{\beta}{c}(s_4 - \frac{\lambda+\delta+\frac{\beta}{2}}{c})^2 T_0 T_{s_4} T_{s_3} T_{s_2} T_{s_1} \gamma_2(0) + 2\frac{\beta}{c} T_0 T_{s_3} T_{s_2} T_{s_1} \gamma_2(0) - T_0 T_{s_3} T_{s_2} T_{s_1} \gamma_1(0). \quad (9)$$

将(8)式和(9)式代入引理 3,并取逆 Laplace 变换得

$$\begin{aligned} m_\delta(u) = {} & \frac{1}{c}\frac{\lambda^2}{c}\int_0^u m_\delta(u-y)((s_4 - \frac{\lambda+\delta}{c})^2 T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_1(y) - 2\frac{\beta}{c}(s_4 - \frac{\lambda+\delta+\frac{\beta}{2}}{c}) T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_2(y) + \\ & 2\frac{\beta}{c} T_{s_3} T_{s_2} T_{s_1} f_2(y) - T_{s_3} T_{s_2} T_{s_1} f_1(y))\mathrm{d}y + \xi_\delta(u) = \\ & \frac{1}{c}\frac{\lambda^2}{c}\int_0^u m_\delta(u-y)((s_4 - \frac{\lambda+\delta}{c})^2 \cdot T_0 T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_1(0) \frac{T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_1(y)}{T_0 T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_1(0)} - \\ & 2\frac{\beta}{c}(s_4 - \frac{\lambda+\delta+\frac{\beta}{2}}{c}) T_0 T_{s_4} T_{s_3} T_{s_2} T_{s_1} \cdot f_2(0) \frac{T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_2(y)}{T_0 T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_2(0)} + 2\frac{\beta}{c} T_0 T_{s_3} T_{s_2} T_{s_1} \cdot \\ & f_2(0) \frac{T_{s_3} T_{s_2} T_{s_1} f_2(y)}{T_0 T_{s_3} T_{s_2} T_{s_1} f_2(0)} - T_0 T_{s_3} T_{s_2} T_{s_1} \cdot f_1(0) \frac{T_{s_3} T_{s_2} T_{s_1} f_1(y)}{T_0 T_{s_3} T_{s_2} T_{s_1} f_1(0)})\mathrm{d}y + \xi_\delta(u) = K_\delta \int_0^u m_\delta(u- \\ & y)((q_{1,\delta} \frac{T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_1(y)}{T_0 T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_1(0)} + q_{2,\delta} \frac{T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_2(y)}{T_0 T_{s_4} T_{s_3} T_{s_2} T_{s_1} f_2(0)} + q_{3,\delta} \frac{T_{s_3} T_{s_2} T_{s_1} f_2(y)}{T_0 T_{s_3} T_{s_2} T_{s_1} f_2(0)} + \\ & (1 - q_{1,\delta} - q_{2,\delta} - q_{3,\delta}) \frac{T_{s_3} T_{s_2} T_{s_1} f_1(y)}{T_0 T_{s_3} T_{s_2} T_{s_1} f_1(0)})\mathrm{d}y + \\ & \xi_\delta(u) = K_\delta \int_0^u m_\delta(u-y) g_\delta(y) dy + \xi_\delta(u). \end{aligned}$$

特别地,当 $\delta = 0$ 时,由引理 2 知,$\tilde{h}_{1,0}(s) - \tilde{h}_{2,0}(s) = 0$ 在有界区域 $D_0 = \{s: Z_0(s) < 1\}$ 内存在 3 个复根,$s_1 = s_1(0), s_2 = s_2(0), s_3 = s_3(0)$ 和一个实根 $s_4 = s_4(0) = 0$.又由于 $s_4 = 0$,从而更新方程

$$m_0(s) = K_0 \int_0^u m_0(u-y) g_0(y) \mathrm{d}y + \xi_0(u)$$

的形式也会更加简单,具体的表达式只需将 $s_4 = s_4(0) = 0$ 代入即可.

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科学家研制新型超密磁带可存储 35TB 数据

从 10 亿 Facebook 用户的更新信息到世界各地医疗机构共享的图片再到高清晰视频流的兴起,人们对海量数据存储的需求不断增长。一直以来,硬盘都充当“驮马”的角色,“背负”大量数据。现在,日本富士胶片公司和瑞士苏黎世的研究人员研发出一种新型超密磁带,被称之为“线性磁带文件系统”。研究人员研制的原型超密磁带覆盖钡铁氧体颗粒图层,所使用的带盒长 10cm,宽 10cm,高 2cm,能够存储 35TB 数据,大约相当于 3500 万本图书所涵盖的信息。线性磁带文件系统的访问速度与硬盘不相上下,比硬盘存储密度更高,能耗更低,能够取代当前的硬盘。线性磁带文件系统可能首先用于世界上最大的射电望远镜阵列平方公里阵列。

(据科学网)