DOI:10.13656/j.cnki.gxkx.2011.04.017

广西科学 Guangxi Sciences 2011,18(4):342～344

NA 随机变量序列加权和的完全收敛性*

Complete Convergence of Weighted Sums of NA Random Variables Sequence

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摘要:利用截尾法和矩不等式,证明一般情况下 NA 随机变量序列加权和的完全收敛性,推广独立随机序列加权和的完全收敛性.

关键词:NA 序列　加权和　完全收敛性

中图法分类号:O211.4　**文献标识码**:A　**文章编号**:1005-9164(2011)04-0342-03

Abstract:The almost sure convergence of the weighted sums of NA random variables sequence was studied by means of truncation and G inequality Lemma. We proved the complete convergence of the weighted sums in the case where the weinghted coefficients α_{nk} are array of real numbers,which extends the convergence result of independent random variable sequence.

Key words:NA sequence,weighted sums,complete convergence

1983 年 Joag-Dev 等[1]提出了 NA 随机变量序列,由于该序列在可靠性理论、渗透理论和多元统计分析理论等均有广泛应用,从而引起了人们的广泛兴趣,现已报道不少成果.1992 年 Matula[2]讨论 NA 序列的几乎处处收敛性,1999 年吴群英[3]研究了同分布 NA 序列的完全收敛性.对于独立同分布随机变量序列,文献[4～6]分别讨论其各种加权和的强收敛性.在许宝禄和 Robbins 引入完全收敛性概念以后,该序列在 iid 情形下的收敛性已被解决得相当完美.本文利用 NA 随机变量序列的截尾法以及矩不等式,把独立情形加权和的完全收敛性推广到 NA 序列.与文献[7]研究的 NA 序列成果相比,本文所得结果在条件上有所放宽,而且更合理.

1　定义及引理

定义 1　称随机变量 $X_1,X_2,\cdots,X_n,n\geqslant 2$ 是 Negatively Associated(简记为 NA)的,若对 $\{1,2,\cdots,n\}$ 的任意两个非空不交子集 A_1,A_2,均有

$$\operatorname{cov}(f_1(X_i;i\in A_1),f_2(X_j;j\in A_2))\leqslant 0,$$

其中 $f_i,i=1,2$,是使上式有意义且对各变元不降(或不升)的函数.

称随机变量列 $\{X_n;n\geqslant 1\}$ 是 NA 列,如果对任意的 $n\geqslant 2,X_1,X_2,\cdots,X_n$ 是 NA 的.

引理 1[1]　设 $\{X_n,n\geqslant 1\}$ 是 NA 随机变量序列,$\forall m\geqslant 2,A_1,A_2,\cdots,A_m$ 是集合 $\{1,2,\cdots,n\}$ 的两两不交的非空子集.如果 $f_i,i=1,2,\cdots,m$ 是对每个变元都非降(或非升)的函数,则

(1) $f_1(X_j;j\in A_1),\cdots,f_m(X_j;j\in A_m)$ 仍是 NA 的.

(2) 如果 $f_i\geqslant 0,i=1,2,\cdots,m$,则还有

$$E(\prod_{i=1}^{m}f_i(X_j;j\in A_i))<\prod_{i=1}^{m}Ef_i(X_j;j\in A_i).$$

(3) 特别的,对任意 $x_i\in R,i=1,2,\cdots,m$,有

$$P(X_1<x_1,\cdots,X_m<x_m)\leqslant\prod_{i=1}^{m}P(X_i<x_i).$$

引理 2[8] 设 $\alpha p>1,1/2<\alpha\leqslant 1,\{X_n,n\geqslant 1\}$ 是独立同分布的随机变量序列,且 $EX_i=0$.则有下列等式等价:

(1) $E|X_1|^p<\infty$;

收稿日期:2011-02-27

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*广西自然科学基金项目(桂科自 2011GXNSFA018149),广西科学基金项目(桂科自 0832262)资助。

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(2) $\sum_{n=1}^{\infty} n^{p\alpha-2} P(\max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} X_i \right| > \varepsilon n^{\alpha}) < \infty$，$\forall \varepsilon > 0$.

引理 3[9] **设** $\alpha p > 1, 1/2 < \alpha \leqslant 1, \{X_n, n \geqslant 1\}$ **是同分布的 NA 随机变量序列，且** $EX_i = 0$. **则下列等式等价：**

(1) **若** $E|X_1|^p < \infty$；

(2) $\sum_{n=1}^{\infty} n^{p\alpha-2} P(\max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} X_i \right| > \varepsilon n^{\alpha}) < \infty$，$\forall \varepsilon > 0$.

引理 4 **令** $\{X, X_n; n \geqslant 1\}$ **是 NA 随机变量序列，**$EX = 0$ **且** $E|X|^p < \infty$. **则存在着一个与** n **无关的常数** D，**对任意** $n \geqslant 1$，

$$E(\max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} X_i \right|^p) \leqslant D\{\sum_{i=1}^{n} E|X_i|^p + (\sum_{i=1}^{n} EX_i^2)^{p/2}\}. \quad (1)$$

2 主要结果

定理 1 **令** $\{X, X_n; n \geqslant 1\}$ **是同分布 NA 随机变量序列，当** $\alpha p > 1, 1/2 < \alpha \leqslant 1$ **时，**$EX = 0$ **且** $E|X|^p < \infty$. **假设** $\{a_{ni}, 1 < i \leqslant n, n \geqslant 1\}$ **是满足** $|a_{ni}| \leqslant 1$，$1 < i \leqslant n, n \geqslant 1$ **的一组数列. 则**

$$\sum_{n=1}^{\infty} n^{p\alpha-2} P(\max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} X_i \right| > \varepsilon n^{\alpha}) < \infty, \forall \varepsilon > 0. \quad (2)$$

证明 当 $1 < i \leqslant n$ 且 $n \geqslant 1$ 时，令 $X'_{ni} = n^{\alpha} I(X_i > n^{\alpha}) + X_i I(|X_i| \leqslant n^{\alpha}) - n^{\alpha} I(X_i < -n^{\alpha})$. 因为 $EX_i = 0$，$\sum_{i=1}^{n} |a_{ni}| \leqslant n$，有

$$n^{-\alpha} \max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} EX'_{ni} \right| = n^{-\alpha} \max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} E(n^{\alpha} I(X_i > n^{\alpha}) + X_i I(|X_i| \leqslant n^{\alpha}) - n^{\alpha} I(X_i < -n^{\alpha})) \right| \leqslant n^{-\alpha} \max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} E(n^{\alpha} I(|X_i| > n^{\alpha}) + X_i I(|X_i| \geqslant n^{\alpha})) \right| \leqslant$$

$$n^{-\alpha} \max_{1\leqslant j\leqslant n} (\sum_{i=1}^{j} a_{ni} E|X_1| I(|X_1| > n^{\alpha}) + \sum_{i=1}^{j} a_{ni} E|X_1| I(|X_1| \geqslant n^{\alpha})) \leqslant Cn^{-\alpha} \sum_{i=1}^{n} |a_{ni}| E|X_1| I(|X_1| > n^{\alpha}) \leqslant Cn^{1-\alpha} E|X_1| I(|X_1| > n^{\alpha}) \leqslant Cn^{1-p\alpha} E|X_1|^p I(|X_1| > n^{\alpha}) \to 0, n \to \infty. \quad (3)$$

所以，n **足够大时，有**

$$n^{-\alpha} \max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} EX'_{ni} \right| < \frac{\varepsilon}{2}. \quad (4)$$

还需要证明

$$\sum_{n=1}^{\infty} n^{p\alpha-2} P(\max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} X_i \right| > \varepsilon n^{\alpha}) \leqslant \sum_{n=1}^{\infty} n^{p\alpha-2} \sum_{i=1}^{n} P(|X_i| > n^{\alpha}) + \sum_{n=1}^{\infty} n^{p\alpha-2} P(\max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} X'_{ni} \right| > \varepsilon n^{\alpha}) \leqslant C\sum_{n=1}^{\infty} n^{p\alpha-1} P(|X| > n^{\alpha}) + C\sum_{n=1}^{\infty} n^{p\alpha-2} P(\max_{1\leqslant j\leqslant n} \left| \sum_{i=1}^{j} a_{ni} (X'_{ni} - \mathrm{E}X'_{ni}) \right| > \frac{\varepsilon n^{\alpha}}{2}) =: I + CJ.$$

因为 $\sum_{n=1}^{\infty} n^{p\alpha-1} P(|X| > n^{\alpha}) \leqslant CE|X|^p < \infty$，**故有** $I < \infty$. **所以只需证明** $J < \infty$ **即可.**

对于任意的 $r \geqslant 2$，**由** Markov **不等式及引理** 1.4 **得**

$$J \leqslant (\frac{2}{\varepsilon})^r \sum_{n=1}^{\infty} n^{p\alpha-r\alpha-2} E(\max_{1\leqslant j\leqslant n} | \sum_{i=1}^{j} a_{ni} (X'_{ni} - EX'_{ni}) |^r \leqslant C\sum_{n=1}^{\infty} n^{p\alpha-r\alpha-2} \{(\sum_{i=1}^{n} a_{ni}^2 E|X'_{ni}|^2)^{r/2} + \sum_{i=1}^{n} |a_{ni}|^r E|X'_{ni}|^r\} \leqslant C\sum_{n=1}^{\infty} n^{p\alpha-r\alpha-2} \cdot$$

$$(\sum_{i=1}^{n} a_{ni}^2 E|n^{\alpha} I(X_{ni} > n^{\alpha}) + X_i I(|X_i| \leqslant n^{\alpha}) - n^{\alpha} I(X_{ni} < -n^{\alpha})|^2)^{r/2} + C\sum_{n=1}^{\infty} n^{p\alpha-r\alpha-2} \cdot$$

$$\sum_{i=1}^{n} |a_{ni}|^r E|n^{\alpha} I(X_{ni} > n^{\alpha}) + X_i I(|X_i| \leqslant n^{\alpha}) - n^{\alpha} I(X_{ni} < -n^{\alpha})|^r \leqslant C\sum_{n=1}^{\infty} n^{p\alpha-r\alpha-2} \cdot$$

$$(\sum_{i=1}^{n} a_{ni}^2 E(|X_i| I(|X_i| \leqslant n^{\alpha}) + n^{\alpha} I(|X_i| \geqslant n^{\alpha}))^2)^{r/2} + C\sum_{n=1}^{\infty} n^{p\alpha-r\alpha-2} \sum_{i=1}^{n} |a_{ni}|^r E|n^{\alpha} I(|X_i| > n^{\alpha}) + X_i I(|X_i| \leqslant n^{\alpha})|^r \leqslant C\sum_{n=1}^{\infty} n^{p\alpha-r\alpha-2+r/2} E|X_1|^p +$$

$$C\sum_{n=1}^{\infty} n^{p\alpha-r\alpha-1} ((E|n^{\alpha}|^r I(|X_i| > n^{\alpha}) + E|X_i|^r I(|X_i| \leqslant n^{\alpha})) =: CJ_1 + CJ_2.$$

当 $p \geqslant 2$，**取** $r > \max\{(p\alpha-1)/(\alpha-1/2), p\}$，**所以** $r > (p\alpha-1)/(\alpha-1/2)$.

因为 $r(\frac{1}{2}-\alpha) < 0$，$p\alpha - 2 < -1$，**所以** $p\alpha -$

$r\alpha+\frac{r}{2}-2<-1$. 所以 $J_1\leqslant C\sum_{n=1}^{\infty}n^{p\alpha-r\alpha-2+r/2}<\infty$,

$$J_2=\sum_{n=1}^{\infty}n^{p\alpha-r\alpha-1}((E|n^{\alpha}|^{r}I(|X_i|>n^{\alpha})+ E|X_i|^{r}I(|X_i|\leqslant n^{\alpha}))\leqslant \sum_{n=1}^{\infty}n^{p\alpha-r\alpha-1}E|X_1|^{r}I(|X_1|>n^{\alpha})+ \sum_{n=1}^{\infty}n^{p\alpha-r\alpha-1}\mathrm{E}|X_1|^{r}I(|X_1|\leqslant n^{\alpha})=:J_3+J_4.$$

因为 $p\alpha-r\alpha-2+\frac{r}{2}<-1$,得 $p\alpha-r\alpha-1<-\frac{r}{2}<-1$,所以

$$J_3=\sum_{n=1}^{\infty}n^{p\alpha-r\alpha-1}E|X_1|^{p}I(|X_1|>n^{\alpha})<\infty,$$

$$J_4=\sum_{n=1}^{\infty}n^{p\alpha-r\alpha-1}\sum_{i=1}^{n}E|X_1|^{r}I((i-1)^{\alpha}<|X_1|\leqslant i^{\alpha})=\sum_{i=1}^{\infty}E|X_1|^{r}I((i-1)^{\alpha}<|X_1|\leqslant i^{\alpha})\sum_{n=1}^{\infty}n^{p\alpha-r\alpha-1}\leqslant C\sum_{i=1}^{\infty}E|X_1|^{r}I((i-1)^{\alpha}<|X_1|\leqslant i^{\alpha})i^{p\alpha-r\alpha}\leqslant CE|X_1|^{p}<\infty.$$

当 $p<2$ 时,取 $r=2$. 因为 $r>p$,上式仍然成立,此时 $J_1=J_2<\infty$.

综上所述,定理 1 证明完毕.

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