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一类具有脉冲和非单调功能反应的扩散时滞捕食系统的多重正周期解*

Multiple Positive Periodic Solutions for a Class of Predator-prey System with Diffusion and Time Delay and Non-monotone Functional Response and Impulsive Effect

周优军[1,2],秦发金[2],姚晓洁[2],曹 亮[1]
ZHOU You-jun[1,2], QIN Fa-jin[2], YAO Xiao-jie[2], CAO Liang[1]

(1.广西大学数学与信息科学学院,广西南宁 530004; 2.柳州师范高等专科学校数学与计算机科学系,广西柳州 545004)
(1. College of Mathematics and Information Science, Guangxi University, Nanning, Guangxi, 530004, China; 2. Department of Mathematics and Computer Science, Liuzhou Teachers College, Liuzhou, Guangxi, 545004, China)

摘要: 利用重合度理论,得到一类具有脉冲和非单调功能反应的扩散时滞捕食系统至少存在4个正周期解的充分条件.

关键词: 微分方程 捕食系统 脉冲效应 功能反应 周期解 重合度
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Abstract Using the coincidence degree theory, the sufficient condition is obtained for the predator-prey diffusion system with the impulsive effect and the non-monotone functional response have at least four positive periodic solutions.

Key words differential equation, predator-prey system, impulsive effect, functional response, periodic solutions, coincidence degree

近年来,关于种群动力系统具有多个正周期解的研究,已经取得了不少的结果[1～5].对种群生态学而言,脉冲效应是经常存在的,比如在固定的时间点,人为地对系统投放或收获,这种投放和收获对系统的数量构成一种脉冲;又如许多种群的出生是季节性的,也可以把这些种群的出生看成是对系统的脉冲.因此,为了更精确地描述种群系统,可以考虑使用脉冲微分方程来研究其各种性态.目前,对具有脉冲作用的动力系统模型周期解的研究已有不少结果[6～9].文献[6]研究了具有脉冲和HollingII类功能反应的扩散时滞竞争系统

$$\begin{cases} x_1'(t) = x_1(t)\left[-d_1(t) - b_1(t)x_1(t) - \dfrac{c_1(t)y(t)}{1+m(t)x_1(t)}\right] + D_1(t)\left[x_2(t) - x_1(t)\right], \\ x_2'(t) = x_2(t)\left[-d_2(t) - b_2(t)x_2(t) - c_2(t)\displaystyle\int_{-\tau}^{0} k(s)x_2(t+s)\,\mathrm{d}s\right] + D_2(t)\left[x_1(t) - x_2(t)\right], \\ y'(t) = y(t)\left[-d_3(t) - b_3(t)y(t) - \dfrac{c_3(t)x_1(t)}{1+m(t)x_1(t)}\right], \\ \qquad t \neq t_k, k \in N; \\ \Delta x_1(t_k) = x_1(t_k^+) - x_1(t_k^-) = b_{1k}x_1(t_k), \\ \Delta x_2(t_k) = x_2(t_k^+) - x_2(t_k^-) = b_{2k}x_2(t_k), \\ \Delta y(t_k) = y(t_k^+) - y(t_k^-) = b_{3k}y(t_k), \\ \qquad t = t_k, k \in N. \end{cases} \quad (1)$$

正周期解的存在性,利用重合度理论获得了系统(1)

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作者简介: 周优军(1974-),男,硕士,主要从事供应链模型与优化,生物数学和智能优化算法研究
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至少存在 1 个正周期解的充分条件. 本文在文献[5,6]的基础上,研究一类具有脉冲和非单调功能反应的扩散时滞捕食系统

$$\begin{cases}\left.\begin{aligned}&x_1'(t)=x_1(t)[-d_1(t)-b_1(t)x_1(t)-\\&\quad\frac{c_1(t)y_1^2(t)}{m^2y_1^2(t)+x_1^2(t)}-\frac{e_1(t)y_2^2(t)}{n^2y_2^2(t)+x_1^2(t)}]+\\&\quad D_1(t)[x_2(t)-x_1(t)],\\&x_2'(t)=x_2(t)[-d_2(t)-b_2(t)x_2(t)-\\&\quad c_2(t)\int_{-\tau}^{0}k(s)x_2(t+s)\mathrm{d}s]+\\&\quad D_2(t)[x_1(t)-x_2(t)],\\&y_1'(t)=y_1(t)[-d_3(t)+\frac{c_3(t)x_1(t)y_1(t)}{m^2y_1^2(t)+x_1^2(t)}],\\&y_2'(t)=y_2(t)[-d_4(t)+\frac{c_4(t)x_1(t)y_2(t)}{n^2y_2^2(t)+x_1^2(t)}],\end{aligned}\right\}\\\quad t\neq t_k,k\in N;\\\left.\begin{aligned}&\Delta x_1(t_k)=x_1(t_k^+)-x_1(t_k^-)=b_{1k}x_1(t_k),\\&\Delta x_2(t_k)=x_2(t_k^+)-x_2(t_k^-)=b_{2k}x_2(t_k),\\&\Delta y_1(t_k)=y_1(t_k^+)-y_1(t_k^-)=b_{3k}y_1(t_k),\\&\Delta y_2(t_k)=y_2(t_k^+)-y_2(t_k^-)=b_{4k}y_2(t_k),\end{aligned}\right\}\\\quad t=t_k,k\in N.\end{cases}\tag{2}$$

具有多个正周期解的充要条件,其中 $b_{ik}x_i(t_k)$ 和 $b_{jk}y_j(t_k)$ 分别表示种群 $x_i(i=1,2)$ 和 $y_j(j=1,2)$ 在 t_k 时刻脉冲出生种群的增加值,$0<b_{ik}<1(i=1,2,3,4)$ $x_j(t_k^+)$,$x_j(t_k^-)$ 和 $y_j(t_k^+)$,$y_j(t_k^-)$ $(j=1,2)$ 分别表示 x_j,y_j 在时刻 t_k 的右极限和左极限,$m,n>0$ 是正常数.

本文总假设:

1)$e_1(t),c_i(t),d_i(t),b_j(t),D_j(t),i=1,2,3,4,j=1,2$,都是正的连续 ω- 周期函数;

2)$k(s)$ 是 $[-\tau,0](0<\tau<\infty)$ 非负连续函数并且 $\int_{-\tau}^{0}k(s)\mathrm{d}s=1$;

3) 存在正整数 p,使得 $t_{k+p}=t_k+\omega,b_{i(k+p)}=b_{ik}$,$i=1,2,3,4$.

1 基本引理

设 $J\subset R$,记 $PC(J,R)$ 是满足以下条件的函数集合:数 $\Psi:J\to R$ 在 $t\in J,t\neq\tau_k$ 处连续,$\tau_k\in J$ 是函数的第一类不连续点且该点左极限存在. 记 $PC^1(J,R)$ 是满足 $\Psi:J\to R$ 且导数 $\frac{\mathrm{d}\psi}{\mathrm{d}t}\in PC(J,R)$ 的函数集合. ω- 周期函数构成 Banach 空间:

$PC_\omega=\{\Psi\in PC([0,\omega],R)|\Psi(0)=\Psi(\omega)\}(\|\Psi\|_{PC_\omega}=\sup\{|\Psi(t)|:t\in[0,\omega]\})$,

$PC_\omega^1=\{\Psi\in PC^1([0,\omega],R)|\Psi(0)=\Psi(\omega)\}(\|\Psi\|_{PC_\omega^1}=\max\{\|\Psi(t)\|_{PC_\omega},\|\Psi'(t)\|_{PC_\omega^1}\})$.

定义 1 集合 A 在 $[0,\omega]$ 上是拟等度连续的,如果对任意 $\varepsilon>0$,存在 $\delta>0$,使得当 $x\in A,k\in N,t_1$,$t_2\in(\tau_{k-1},\tau_k]\cap[0,\omega]$,且 $|t_1-t_2|<\delta$ 时,有 $|x(t_1)-x(t_2)|<\varepsilon$.

引理 1[10](紧性判别)集合 $A\subset PC_\omega$ 是相对紧的当且仅当:1)A 是有界的,即对每一个 $x\in A$ 和某些 $M>0$,有 $\|\Psi\|_{PC_\omega}=\sup\{|\Psi(t)|:t\in[0,\omega]\}\leqslant M$;2)$A$ 在集合 $[0,\omega]$ 上是拟等度连续的.

重合度理论:设 X,Z 是赋范向量空间,$L:\mathrm{Dom}L\subset X\to Z$ 为线性映射,$N:X\times[0,1]\to Z$ 为连续映射,如果 $\dim\mathrm{Ker}L=\mathrm{codim}\ \mathrm{Im}L<+\infty$,且 $\mathrm{Im}L$ 为 Z 中的闭子集,则映射 L 称为零指标的 Fredholm 映射. 如果 L 是零指标的 Fredholm 映射,且存在连续投影 $P:X\to X$ 及 $Q:Z\to Z$ 使得 $\mathrm{Im}P=\mathrm{Ker}L,\mathrm{Im}L=\mathrm{Ker}Q=\mathrm{Im}(I-Q)$,则 $L|_{\mathrm{Dom}L\cap\mathrm{Ker}P}:(I-P)X\to\mathrm{Im}L$ 可逆,并设其逆映射为 K_P. 设 Ω 为 X 中有界开集,如果 $QN(\bar{\Omega}\times[0,1])$ 有界且 $K_P(I-Q)N:\bar{\Omega}\times[0,1]\to X$ 是紧的,则称 N 在 $\bar{\Omega}\times[0,1]$ 上是 L- 紧的. 由于 $\mathrm{Im}Q$ 与 $\mathrm{Ker}L$ 同构,因而存在同构映射 $J:\mathrm{Im}Q\to\mathrm{Ker}L$.

引理 2[11](Mawhin 延拓定理) 设 L 是指标为零的 Fredholm 映射,N 在 $\bar{\Omega}\times[0,1]$ 是 L- 紧的,假设:1) 对任意的 $\lambda\in(0,1)$,方程 $Lx=\lambda N(x,\lambda)$ 的解满足 $x\notin\partial\Omega$;2)$QN(x,\lambda)\neq0,\forall x\in\partial\Omega\cap\mathrm{Ker}L$;3)$\deg\{JQN(.,0),\Omega\cap\mathrm{Ker}L,0\}\neq0$. 则方程 $Lx=Nx$ 在 $\mathrm{Dom}L\cap\bar{\Omega}$ 内至少有一个解.

引理 3 $R_+^4=\{(x_1,x_2,y_1,y_2)|x_i>0,y_i>0,i=1,2\}$ 是系统(2) 关于正初值的一个正不变集.

设函数 $f(t)\in C_\omega$ 或 PC_ω,记 $\bar{f}=(1/\omega)\int_0^\omega f(t)\mathrm{d}t$,并且令

$$r_i=(1/\omega)\sum_{k=1}^{p}\ln(1+b_{ik})-\bar{d}_j,r_j=\bar{d}_j-(1/\omega)\sum_{k=1}^{p}\ln(1+b_{jk}),i=1,2,j=3,4.$$

假设下列条件满足:

$(\mathrm{H}_1)r_1>\frac{\bar{c}_1}{m^2}+\frac{\bar{e}_1}{n^2}$;$(\mathrm{H}_2)r_2>\bar{c}_2e^{B_2}$;$(\mathrm{H}_3)r_3>0,r_4>0$;$(\mathrm{H}_4)e^{E_1-2\bar{d}_3\omega}\bar{c}_3>2r_3me^{B_1}$;$(\mathrm{H}_5)e^{E_1-2\bar{d}_4\omega}\bar{c}_4>2r_4ne^{B_1}$.

这里 $B_i=\max\{\ln\frac{r_1}{\bar{b}_1},\ln\frac{r_2}{\bar{b}_2}\}+2\sum_{k=1}^{p}\ln(1+b_{ik}),i=1,2,E_1=\min\{\frac{r_1-\frac{\bar{c}_1}{m^2}}{\bar{b}_1},\ln\frac{r_2-\bar{c}_2e^{B_2}}{\bar{b}_2}\}\ln-2\sum_{k=1}^{p}\ln(1+b_{ik})$.

引入 12 个正数

$$l_{1\pm}=\frac{e^{B_1+2\bar{d}_3\omega\bar{c}_3}\pm\sqrt{(e^{B_1+2\bar{d}_3\omega\bar{c}_3})^2-4r_3^2m^2e^{2E_1}}}{2r_3m^2},$$

$$u_{1\pm}=\frac{e^{E_1-2\bar{d}_3\omega\bar{c}_3}\pm\sqrt{(e^{E_1-2\bar{d}_3\omega\bar{c}_3})^2-4r_3^2m^2e^{2B_1}}}{2r_3m^2},$$

$$l_{2\pm}=\frac{e^{B_1+2\bar{d}_4\omega\bar{c}_4}\pm\sqrt{(e^{B_1+2\bar{d}_4\omega\bar{c}_4})^2-4r_4^2n^2e^{2E_1}}}{2r_4n^2}$$

$$u_{2\pm}=\frac{e^{E_1-2\bar{d}_4\omega\bar{c}_4}\pm\sqrt{(e^{E_1-2\bar{d}_4\omega\bar{c}_4})^2-4r_4^2n^2e^{2B_1}}}{2r_4n^2},$$

$$x_{1\pm}=\frac{\frac{r_1\bar{c}_3}{\bar{b}_1}\pm\sqrt{(\frac{r_1\bar{c}_3}{\bar{b}_1})^2-4r_3^2m^2(\frac{r_1}{\bar{b}_1})^2}}{2r_3m^2},$$

$$x_{2\pm}=\frac{\frac{r_1\bar{c}_4}{\bar{b}_1}\pm\sqrt{(\frac{r_1\bar{c}_4}{\bar{b}_1})^2-4r_4^2n^2(\frac{r_1}{\bar{b}_1})^2}}{2r_4n^2}.$$

显然有

$$l_{i-}<x_{i-}<u_{i-}<u_{i+}<x_{i+}<l_{i+},i=1,2.$$

2 系统正周期解的存在性

定理1 如果条件$(H_1)\sim(H_5)$成立，则系统(2)至少存在4个正ω-周期解.

证明 对系统(2)作变换：$x_i(t)=e^{u_i(t)},i=1,2$，$y_1(t)=e^{u_3(t)},y_2(t)=e^{u_4(t)}$，有

$$\begin{cases}\left.\begin{aligned}u_1'(t)&=-d_1(t)-b_1(t)e^{u_1(t)}-\\&\quad\frac{c_1(t)e^{2u_3(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}}-\frac{e_1(t)e^{2u_4(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}}+\\&\quad D_1(t)e^{u_2(t)-u_1(t)}-D_1(t),\\u_2'(t)&=-d_2(t)-b_2(t)e^{u_2(t)}-\\&\quad c_2(t)\int_{-\tau}^0k(s)e^{u_2(t+s)}ds+\\&\quad D_2(t)e^{u_1(t)-u_2(t)}-D_2(t),\\u_3'(t)&=-d_3(t)+\frac{c_3(t)e^{u_1(t)+u_3(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}},\\u_4'(t)&=-d_4(t)+\frac{c_4(t)e^{u_1(t)+u_4(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}},\end{aligned}\right\}\quad t\neq t_k,k\in N;\\\left.\begin{aligned}\Delta u_1(t_k)&=\ln(1+b_{1k}),\\\Delta u_2(t_k)&=\ln(1+b_{2k}),\\\Delta u_3(t_k)&=\ln(1+b_{3k}),\\\Delta u_4(t_k)&=\ln(1+b_{4k}),\end{aligned}\right\}t=t_k,k\in N.\end{cases}\tag{3}$$

如果系统(3)存在一个ω-周期解$(u_1^*,u_2^*,u_3^*,u_4^*)^T$，则$(e^{u_1^*},e^{u_2^*},e^{u_3^*},e^{u_4^*})^T$为系统(2)的一个正$\omega$-周期解.

记$\mathrm{Dom}L=PC_\omega^1\times PC_\omega^1\times PC_\omega^1\times PC_\omega^1$，

$$L:\mathrm{Dom}L\to Z,\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix}\to\left(\begin{pmatrix}u_1'\\u_2'\\u_3'\\u_4'\end{pmatrix},\left\{\begin{pmatrix}\Delta u_1(t_k)\\\Delta u_2(t_k)\\\Delta u_3(t_k)\\\Delta u_4(t_k)\end{pmatrix}\right\}_{k=1}^p\right),$$

$N:PC_\omega^1\times PC_\omega^1\times PC_\omega^1\times PC_\omega^1\times[0,1]\to Z$满足

$$N\left(\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix},\lambda\right)=\left(\begin{pmatrix}N_1(t)\\N_2(t)\\N_3(t)\\N_4(t)\end{pmatrix},\left\{\begin{pmatrix}\ln(1+b_{1k})\\\ln(1+b_{2k})\\\ln(1+b_{3k})\\\ln(1+b_{4k})\end{pmatrix}\right\}_{k=1}^p\right),$$

$\lambda\in[0,1]$.

其中

$$N_1(t)=-d_1(t)-b_1(t)e^{u_1(t)}-\frac{\lambda c_1(t)e^{2u_3(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}}-\frac{\lambda e_1(t)e^{2u_4(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}}+\lambda D_1(t)e^{u_2(t)-u_1(t)}-\lambda D_1(t),$$

$$N_2(t)=-d_2(t)-b_2(t)e^{u_2(t)}-c_2(t)\int_{-\tau}^0k(s)e^{u_2(t+s)}ds+\lambda D_2(t)e^{u_1(t)-u_2(t)}-\lambda D_2(t),$$

$$N_3(t)=-d_3(t)+\frac{c_3(t)e^{u_1(t)+u_3(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}},$$

$$N_4(t)=-d_4(t)+\frac{c_4(t)e^{u_1(t)+u_4(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}}.$$

显然有

$$\mathrm{Ker}L=\left\{\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix}:\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix}=\begin{pmatrix}h_1\\h_2\\h_3\\h_4\end{pmatrix}\in R^4,t\in[0,\omega]\right\},$$

$$\mathrm{Im}L=$$

$$\left\{z=\left(\begin{pmatrix}f_1\\f_2\\f_3\\f_4\end{pmatrix},\left\{\begin{pmatrix}a_k\\b_k\\c_k\\d_k\end{pmatrix}\right\}_{k=1}^p\right)\in Z:\begin{pmatrix}\int_0^\omega f_1dt+\sum_{k=1}^pa_k=0\\\int_0^\omega f_2dt+\sum_{k=1}^pb_k=0\\\int_0^\omega f_3dt+\sum_{k=1}^pc_k=0\\\int_0^\omega f_4dt+\sum_{k=1}^pd_k=0\end{pmatrix}\right\}.$$

由于$\mathrm{Im}L$在Z中是闭的，L是一个指标为零的Fredholm映射. 易知P和Q是连续的投影且使得$\mathrm{Im}P=\mathrm{Ker}L,\mathrm{Ker}P=\mathrm{Im}L=\mathrm{Im}(I-Q)$，其中

$$P\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix}=\frac{1}{\omega}\begin{pmatrix}\int_0^\omega u_1(t)dt+\sum_{k=1}^pa_k\\\int_0^\omega u_2(t)dt+\sum_{k=1}^pb_k\\\int_0^\omega u_3(t)dt+\sum_{k=1}^pc_k\\\int_0^\omega u_4(t)dt+\sum_{k=1}^pd_k\end{pmatrix},$$

$$QZ=Q\left(\begin{pmatrix}f_1\\f_2\\f_3\\f_4\end{pmatrix},\left\{\begin{pmatrix}a_k\\b_k\\c_k\\d_k\end{pmatrix}\right\}_{k=1}^p\right)=$$

$$\left(\frac{1}{\omega}\begin{pmatrix}\int_0^\omega f_1(t)\mathrm{d}t+\sum_{k=1}^p a_k\\ \int_0^\omega f_2(t)\mathrm{d}t+\sum_{k=1}^p b_k\\ \int_0^\omega f_3(t)\mathrm{d}t+\sum_{k=1}^p c_k\\ \int_0^\omega f_4(t)\mathrm{d}t+\sum_{k=1}^p d_k\end{pmatrix},\left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}_{k=1}^p\right).$$

从而，广义逆$(L)K_P$：$\mathrm{Im}L\to \mathrm{Ker}P\cap \mathrm{Dom}L$是

$$K_Pz=$$

$$\begin{pmatrix}\int_0^t f_1\mathrm{d}s+\sum_{t>t_k}a_k-(1/\omega)\int_0^\omega\int_0^t f_1\mathrm{d}s\mathrm{d}t-\sum_{k=1}^p a_k\\ \int_0^t f_2\mathrm{d}s+\sum_{t>t_k}b_k-(1/\omega)\int_0^\omega\int_0^t f_2\mathrm{d}s\mathrm{d}t-\sum_{k=1}^p b_k\\ \int_0^t f_3\mathrm{d}s+\sum_{t>t_k}c_k-(1/\omega)\int_0^\omega\int_0^t f_3\mathrm{d}s\mathrm{d}t-\sum_{k=1}^p c_k\\ \int_0^t f_4\mathrm{d}s+\sum_{t>t_k}d_k-(1/\omega)\int_0^\omega\int_0^t f_4\mathrm{d}s\mathrm{d}t-\sum_{k=1}^p d_k\end{pmatrix}.$$

因此

$$QN\left(\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix},\lambda\right)=\left(\begin{pmatrix}g_1\\g_2\\g_3\\g_4\end{pmatrix},\left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}_{k=1}^p\right),$$

其中

$$g_1=r_1-(1/\omega)\Big[\int_0^\omega b_1(t)e^{u_1}\mathrm{d}t+\int_0^\omega\frac{\lambda c_1(t)e^{2u_3}}{m^2e^{2u_3}+e^{2u_1}}\mathrm{d}t+\int_0^\omega\frac{\lambda e_1(t)e^{2u_4}}{n^2e^{2u_4}+e^{2u_1}}\mathrm{d}t-\lambda\int_0^\omega D_1(t)e^{u_2-u_1}\mathrm{d}t\Big]-\lambda\overline{D}_1,$$

$$g_2=r_2-(1/\omega)\Big[\int_0^\omega b_2(t)e^{u_2}\mathrm{d}t+\int_0^\omega c_2(t)\int_{-\tau}^0 k(s)e^{u_2(t+s)}\mathrm{d}s\mathrm{d}t-\lambda\int_0^\omega D_2(t)e^{u_1-u_2}\mathrm{d}t\Big]-\lambda\overline{D}_2,$$

$$g_3=r_3-(1/\omega)\int_0^\omega\frac{c_3(t)e^{u_1+u_3}}{m^2e^{2u_3}+e^{2u_1}}\mathrm{d}t,$$

$$g_4=r_3-(1/\omega)\int_0^\omega\frac{c_4(t)e^{u_1+u_4}}{n^2e^{2u_4}+e^{2u_1}}\mathrm{d}t.$$

$$K_P(I-Q)N\left(\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix},\lambda\right)=$$

$$\begin{pmatrix}\int_0^t N_1(s)\mathrm{d}s+\sum_{t>t_k}\ln(1+b_{1k})\\ \int_0^t N_2(s)\mathrm{d}s+\sum_{t>t_k}\ln(1+b_{2k})\\ \int_0^t N_3(s)\mathrm{d}s+\sum_{t>t_k}\ln(1+b_{3k})\\ \int_0^t N_4(s)\mathrm{d}s+\sum_{t>t_k}\ln(1+b_{4k})\end{pmatrix}-\begin{pmatrix}(1/\omega)\int_0^\omega\int_0^t N_1(s)\mathrm{d}s\mathrm{d}t+\sum_{k=1}^p\ln(1+b_{1k})\\ (1/\omega)\int_0^\omega\int_0^t N_2(s)\mathrm{d}s\mathrm{d}t+\sum_{k=1}^p\ln(1+b_{2k})\\ (1/\omega)\int_0^\omega\int_0^t N_3(s)\mathrm{d}s\mathrm{d}t+\sum_{t>t_k}\ln(1+b_{3k})\\ (1/\omega)\int_0^\omega\int_0^t N_4(s)\mathrm{d}s\mathrm{d}t+\sum_{t>t_k}\ln(1+b_{4k})\end{pmatrix}-$$

$$\begin{pmatrix}(\frac{t}{\omega}-\frac{1}{2})\int_0^\omega N_1(s)\mathrm{d}s+\sum_{k=1}^p\ln(1+b_{1k})\\ (\frac{t}{\omega}-\frac{1}{2})\int_0^\omega N_2(s)\mathrm{d}s+\sum_{k=1}^p\ln(1+b_{2k})\\ (\frac{t}{\omega}-\frac{1}{2})\int_0^\omega N_3(s)\mathrm{d}s+\sum_{k=1}^p\ln(1+b_{3k})\\ (\frac{t}{\omega}-\frac{1}{2})\int_0^\omega N_4(s)\mathrm{d}s+\sum_{k=1}^p\ln(1+b_{4k})\end{pmatrix}.$$

显然QN和$K_P(I-Q)N$是连续的. 利用引理1，对任意的有界集$\Omega\subset X$，集合$K_P(I-Q)N$是紧的. 从$\mathrm{Im}Q$到$\mathrm{Ker}L$上的同构映射

$$J:\mathrm{Im}Q\to X,\left(\begin{pmatrix}h_1\\h_2\\h_3\\h_4\end{pmatrix},\left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}_{k=1}^p\right)\to\begin{pmatrix}h_1\\h_2\\h_3\\h_4\end{pmatrix}$$

对算子方程$Lu=\lambda N(u,\lambda)$，$u=(u_1,u_2,u_3,u_4)^{\mathrm{T}}$，$\lambda\in(0,1)$，有

$$\begin{cases}\left.\begin{aligned}&u_1'(t)=\lambda\Big[-d_1(t)-b_1(t)e^{u_1(t)}-\frac{c_1(t)e^{2u_3(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}}-\frac{e_1(t)e^{2u_4(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}}+D_1(t)e^{u_2(t)-u_1(t)}-D_1(t)\Big],\\ &u_2'(t)=\lambda\Big[-d_2(t)-b_2(t)e^{u_2(t)}-c_2(t)\int_{-\tau}^0 k(s)e^{u_2(t+s)}\mathrm{d}s+D_2(t)e^{u_1(t)-u_2(t)}-D_2(t)\Big],\\ &u_3'(t)=\lambda\Big[-d_3(t)+\frac{c_3(t)e^{u_1(t)+u_3(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}}\Big],\\ &u_4'(t)=\lambda\Big[-d_4(t)+\frac{c_4(t)e^{u_1(t)+u_4(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}}\Big],\end{aligned}\right\}\quad t\neq t_k,k\in N;\\ \left.\begin{aligned}&\Delta u_1(t_k)=\lambda\ln(1+b_{1k}),\\ &\Delta u_2(t_k)=\lambda\ln(1+b_{2k}),\\ &\Delta u_3(t_k)=\lambda\ln(1+b_{3k}),\\ &\Delta u_4(t_k)=\lambda\ln(1+b_{4k}),\end{aligned}\right\}\quad t=t_k,k\in N.\end{cases}\tag{4}$$

设$u=(u_1,u_2,u_3,u_4)^{\mathrm{T}}$是(4)式对某个$\lambda\in(0,1)$的$\omega$-周期解，将(4)式进行积分得

$$
\begin{cases}
\int_0^\omega [d_1(t)+b_1(t)e^{u_1(t)}+\frac{\lambda c_1(t)e^{2u_3(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}}+ \frac{\lambda c_1(t)e^{2u_4(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}}-\lambda D_1(t)e^{u_2(t)-u_1(t)}+ \lambda D_1(t)]\mathrm{d}t=\sum_{k=1}^{p}\ln(1+b_{1k}),\\
\int_0^\omega [d_2(t)+b_2(t)e^{u_2(t)}+c_2(t)\int_{-\tau}^{0}k(s)e^{u_2(t+s)}\mathrm{d}s- \lambda D_2(t)e^{u_1(t)-u_2(t)}+\lambda D_2(t)]\mathrm{d}t= \sum_{k=1}^{p}\ln(1+b_{2k}),\\
\int_0^\omega \frac{c_3(t)e^{u_3(t)+u_1(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}}\mathrm{d}t=\bar{d}_3\omega-\sum_{k=1}^{p}\ln(1+b_{3k}),\\
\int_0^\omega \frac{c_4(t)e^{u_4(t)+u_1(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}}\mathrm{d}t=\bar{d}_4\omega-\sum_{k=1}^{p}\ln(1+b_{4k}).
\end{cases} \tag{5}
$$

由(5)式得

$$\int_0^\omega |u_i'(t)|\mathrm{d}t\leqslant 2\sum_{k=1}^{p}\ln(1+b_{ik}),i=1,2, \tag{6}$$

$$\int_0^\omega |u_i'(t)|\mathrm{d}t\leqslant 2\bar{d}_i\omega,i=3,4, \tag{7}$$

定义 $V_1(t)=\max\{u_1(t),u_2(t)\}$，下面分两种情形讨论：

情形1 如果 $u_1(t)\geqslant u_2(t)$，则 $V_1(t)=u_1(t)$，由(5)式第1个方程得 $\int_0^\omega b_1(t)e^{V_1(t)}\mathrm{d}t\leqslant r_1\omega$.

情形2 如果 $u_1(t)\leqslant u_2(t)$，则 $V_1(t)=u_2(t)$，由(5)式第2个方程得 $\int_0^\omega b_2(t)e^{V_1(t)}\mathrm{d}t\leqslant r_2\omega$.

选取 $\zeta_i\in[0,\omega],i=1,2,3,t^*\in[0,\omega]$ 使得 $u_i(\zeta_i)=\min\limits_{t\in[0,\omega]}u_i(t),i=1,2,3,V_1(t^*)=\min\limits_{t\in[0,\omega]}V_1(t)$. 由情形1和情形2可得 $V_1(t^*)\leqslant\max\{\ln\frac{r_1}{\bar{b}_1},\ln\frac{r_2}{\bar{b}_2}\}\triangleq m_1$，则

$$u_i(\zeta_i)\leqslant V_1(t^*)\leqslant m_1,i=1,2. \tag{8}$$

由(6)式和(8)式得

$$u_i(t)\leqslant u_i(\zeta_i)+\int_0^\omega |u_i'(t)|\mathrm{d}t\leqslant m_1+2\sum_{k=1}^{p}\ln(1+b_{ik})=B_i,i=1,2. \tag{9}$$

定义 $V_2(t)=\min\{u_1(t),u_2(t)\}$，也分两种情形讨论：

情形3 如果 $u_1(t)\leqslant u_2(t)$，则 $V_2(t)=u_1(t)$，由(5)式第1个方程得

$$\int_0^\omega b_1(t)e^{u_1(t)}\mathrm{d}t=\sum_{k=1}^{p}\ln(1+b_{1k})-\bar{d}_1\omega-\int_0^\omega[\frac{\lambda c_1(t)e^{2u_3(t)}}{m^2e^{2u_3(t)}+e^{2u_1(t)}}+\frac{\lambda e_1(t)e^{2u_4(t)}}{n^2e^{2u_4(t)}+e^{2u_1(t)}}-\lambda D_1(t)e^{u_2(t)-u_1(t)}+\lambda D_1(t)]\mathrm{d}t\geqslant(r_1-\frac{\bar{c}_1}{m^2}-\frac{\bar{e}_1}{n^2})\omega.$$

情形4 如果 $u_1(t)>u_2(t)$，则 $V_2(t)=u_2(t)$，由(5)式第2个方程和(9)式得

$$\int_0^\omega b_2(t)e^{u_2(t)}\mathrm{d}t=\sum_{k=1}^{p}\ln(1+b_{2k})-\bar{d}_2\omega-\int_0^\omega[c_2(t)\int_{-\tau}^{0}k(s)e^{u_2(t+s)}\mathrm{d}s-\lambda D_2(t)e^{u_1(t)-u_2(t)}+\lambda D_2(t)]\mathrm{d}t\geqslant(r_2-\bar{c}_2e^{B_2})\omega.$$

选取 $\eta_i\in[0,\omega],i=1,2,3,4,t_*\in[0,\omega]$ 使得 $u_i(\eta_i)=\min\limits_{t\in[0,\omega]}u_i(t),i=1,2,3,4,V_2(t_*)=\max\limits_{t\in[0,\omega]}V_2(t)$. 由情形3和情形4可得

$$V_2(t_*)\geqslant\min\{\ln\frac{r_1-\frac{\bar{c}_1}{m^2}-\frac{\bar{e}_1}{n^2}}{\bar{b}_1},\ln\frac{r_2-\bar{c}_2e^{B_2}}{\bar{b}_2}\}\triangleq m_2,$$

那么

$$u_i(\eta_i)\geqslant V_2(t_*)\geqslant m_2,i=1,2. \tag{10}$$

由(6)式和(10)式得

$$u_i(t)\geqslant u_i(\eta_i)-\int_0^\omega|u_i'(t)|\mathrm{d}t\geqslant m_2-2\sum_{k=1}^{p}\ln(1+b_{ik})=E_i,i=1,2. \tag{11}$$

由(9)式和(11)式得

$$\max_{t\in[0,\omega]}|u_i(t)|\leqslant\max\{|B_i|,|E_i|\}\triangleq F_i,i=1,2. \tag{12}$$

由(5)式第3个方程得

$$r_3\omega\leqslant\int_0^\omega\frac{c_3(t)e^{B_1+u_3(\eta_3)}}{m^2e^{2u_3(\zeta_3)}+e^{2E_1}}\mathrm{d}t=\frac{\bar{c}_2\omega e^{B_1+u_3(\eta_3)}}{m^2e^{2u_3(\zeta_3)}+e^{2E_1}},$$

即 $u_3(\eta_3)\geqslant\ln(\frac{r_3m^2e^{2u_3(\zeta_3)}+r_3e^{2E_1}}{\bar{c}_3e^{B_1}})$. 结合(7)式得

$$u_3(t)\geqslant u_3(\eta_3)-\int_0^\omega|u_3'(t)|\mathrm{d}t\geqslant\ln\frac{r_3m^2e^{2u_3(\zeta_3)}+r_3e^{2E_1}}{\bar{c}_3e^{B_1}})-2\bar{d}_3\omega,$$

特别有 $u_3(\zeta_3)\geqslant\ln(\frac{r_3m^2e^{2u_3(\zeta_3)}+r_3e^{2E_1}}{\bar{c}_3e^{B_1}})-2\bar{d}_3\omega$，即

$$r_3m^2e^{2u_3(\zeta_3)}-e^{B_1+2\bar{d}_3\omega}\bar{c}_3e^{u_3(\zeta_3)}+r_3e^{2E_1}\leqslant 0,$$

则有

$$\ln l_{1-}\leqslant u_3(\zeta_3)\leqslant\ln l_{1+}, \tag{13}$$

所以

$$u_3(t)\leqslant u_3(\zeta_3)+\int_0^\omega|u_3'(t)|\mathrm{d}t\leqslant\ln l_{1+}+2\bar{d}_3\omega\triangleq B_3. \tag{14}$$

再由(5)式第2个方程得

$$r_3\omega\geqslant\int_0^\omega\frac{c_3(t)e^{E_1+u_3(\zeta_3)}}{m^2e^{2u_3(\eta_3)}+e^{2B_1}}\mathrm{d}t=\frac{\bar{c}_3\omega e^{E_1+u_3(\zeta_3)}}{m^2e^{2u_3(\eta_3)}+e^{2B_1}},$$

即 $u_3(\zeta_3)\leqslant\ln\frac{r_3m^2e^{2u_3(\eta_3)}+r_3e^{2B_1}}{\bar{c}_3\omega e^{E_1}}$，结合(6)式得

$$u_3(t)\leqslant u_3(\zeta_3)+\int_0^\omega|u_3'(t)|\mathrm{d}t\leqslant$$

$\ln\left(\frac{r_3 m^2 e^{2u_3(\xi_3)} + r_3 e^{2B_1}}{\bar{c}_3 \omega e^{E_1}}\right) + 2\bar{d}_3\omega$,

特别有 $u_3(\xi_3) \leqslant \ln\left(\frac{r_3 m^2 e^{2u_3(\xi_3)} + r_3 e^{2B_1}}{\bar{c}_3 e^{E_1}}\right) + 2\bar{d}_3\omega$,即

$$r_3 m^2 e^{2u_3(\xi_3)} - e^{E_1 - 2\bar{d}_3\omega}\bar{c}_3 e^{2u_3(\xi_3)} + r_3 e^{2B_1} \geqslant 0,$$

则有

$$u_3(\xi_3) \geqslant \ln u_{1+} \text{ 或 } u_3(\xi_3) \leqslant \ln u_{1-}. \quad (15)$$

由(13)式、(14)式和(15)式得

$$u_3(t) \in (\ln l_{1-}, \ln u_{1-}) \text{ 或 } u_3(t) \in (\ln u_{1+}, B_3). \quad (16)$$

由(5)式的第4个方程,再类似(13)式和(15)式的证明可得 $u_4(t) \in (\ln l_{2-}, \ln u_{2-})$ 或 $u_4(t) \in (\ln u_{2+}, B_4)$ 其中 $B_4 = \ln l_{2+} + 2\bar{d}_4\omega$.显然 $l_{\pm}, u_{2\pm}, B_3, B_4$ 的选取与 λ 无关.取 $B_5 > \left|\ln\frac{r_1}{\bar{b}_1}\right| + \left|\ln\frac{r_2}{\bar{b}_2 + \bar{c}_2}\right|$,定义

$$\Omega_1 = \left\{ (u_1, u_2, u_3, u_4)^{\mathrm{T}} \middle| \begin{array}{l} |u_1(t)| < F_1 + B_5, \\ |u_2(t)| < F_2 + B_5, \\ u_3(t) \in (\ln l_{1-}, \ln u_{1-}), \\ u_4(t) \in (\ln l_{2-}, \ln u_{2-}). \end{array} \right\},$$

$$\Omega_2 = \left\{ (u_1, u_2, u_3, u_4)^{\mathrm{T}} \middle| \begin{array}{l} |u_1(t)| < F_1 + B_5, \\ |u_2(t)| < F_2 + B_5, \\ u_3(t) \in (\ln l_{1-}, \ln u_{1-}), \\ u_4(t) \in (\ln u_{2+}, B_4). \end{array} \right\},$$

$$\Omega_3 = \left\{ (u_1, u_2, u_3, u_4)^{\mathrm{T}} \middle| \begin{array}{l} |u_1(t)| < F_1 + B_5, \\ |u_2(t)| < F_2 + B_5, \\ u_3(t) \in (\ln l_{1+}, B_3), \\ u_3(t) \in (\ln l_{2-}, \ln u_{2-}). \end{array} \right\},$$

$$\Omega_4 = \left\{ (u_1, u_2, u_3, u_4)^{\mathrm{T}} \middle| \begin{array}{l} |u_1(t)| < F_1 + B_5, \\ |u_2(t)| < F_2 + B_5, \\ u_3(t) \in (\ln u_{1+}, B_3), \\ u_3(t) \in (\ln u_{2+}, B_4). \end{array} \right\},$$

显然 $\Omega_i \cap \Omega_j = \varnothing, i, j = 1, 2, 3, 4, i \neq j$,因此 $\Omega_i (i = 1, 2, 3, 4)$ 满足引理2的条件1).

考虑代数方程组

$$\begin{cases} r_1 - \bar{b}_1 e^{u_1} = 0, \\ r_2 - \bar{b}_2 e^{u_2} - \bar{c}_2 e^{u_2} = 0, \\ -r_3 + \dfrac{\bar{c}_3 e^{u_3 + u_1}}{m^2 e^{2u_3} + e^{2u_1}} = 0, \\ -r_4 + \dfrac{\bar{c}_4 e^{u_4 + u_1}}{n^2 e^{2u_4} + e^{2u_1}} = 0. \end{cases} \quad (17)$$

显然方程组(17)有4个解

$v_1^* = \left(\ln\frac{r_1}{\bar{b}_1}, \ln\frac{r_2}{\bar{b}_2 + \bar{c}_2}, \ln x_{1-}, \ln x_{2-}\right), v_2^* = \left(\ln\frac{r_1}{\bar{b}_1}, \ln\frac{r_2}{\bar{b}_2 + \bar{c}_2}, \ln x_{1-}, \ln x_{2+}\right), v_3^* = \left(\ln\left(\frac{r_1}{\bar{b}_1}\right), \ln\left(\frac{r_2}{\bar{b}_2 + \bar{c}_2}\right), \ln x_{1+}, \ln x_{2-}\right), v_4^* = \left(\ln\frac{r_1}{\bar{b}_1}, \ln\frac{r_2}{\bar{b}_2 + \bar{c}_2}, \ln x_{1+}, \ln x_{2+}\right)$,并且 $v_i^* \in \Omega_i, i = 1, 2, 3, 4$,则当 $u \in \partial\Omega_i \cap \mathrm{Ker}L = \partial\Omega_i \cap R^2, i = 1, 2$时,有 $QN(u, 0) \neq 0$.直接计算可得 $\deg\{JQN(\cdot, 0), \Omega_i \cap \mathrm{Ker}L, 0\} \neq 0, i = 1, 2, 3, 4$.

根据引理2,系统(3)有4个 ω-周期解 $(u_{1i}^*(t), u_{2i}^*(t), u_{3i}^*(t), u_{4i}^*(t))^{\mathrm{T}}, i = 1, 2, 3, 4$,所以,系统(2)的4个 ω-正周期解为 $(e^{u_{1i}^*(t)}, e^{u_{2i}^*(t)}, e^{u_{3i}^*(t)}, e^{u_{4i}^*(t)})^{\mathrm{T}}, i = 1, 2, 3, 4$.

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