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A Stage-Structured Predator-Prey System with Time Delay and TypeⅢ Functional Response*

一类具有阶段结构时滞功能反应的捕食者-食饵模型

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Abstract A non-autonomous predator-prey model with stage-structured on prey, time delay, type Ⅲ functional response, continuous harvesting on predator has been studied in this paper. The existence of a positive periodic solution of the system has been established.

Key words predator-prey model, stage-structured, typeⅢ functional response, time delay, periodic solution

摘要:研究一类食饵具有阶段结构、捕食者具有时滞功能反应和连续收获的非自治捕食者-食饵模型,得到该模型存在正周期解的充分条件.

关键词:捕食者模型 阶段结构 HollingⅢ类功能反应 时滞 周期解
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The research on biology models becomes meaningful in order to exploit biological resources reasonably. As we all know, physiological function (birth rate, Morality, Diffusivity, Predation Capacity etc.) of species varies in different stages. Moreover, there is interaction between mature and immature species. All of these affect the continuous existence and extinction of biological species more or less. Therefore, it is practically meaningful to study stage-structured biological species models[1].

Stage-structured models have received much attention in recent years. Mathematical analyses for stage-structured models were demonstrated in many papers[2~7]. Wang and Chen[2] considered the following predator-prey models:

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$$
\begin{cases}
\dot{x}_1(t) = x_1(t)[r - ax_1(t-\tau_1) - bx_3(t)], \\
\dot{x}_2(t) = kbx_1(t-\tau_2)x_3(t-\tau_2) - \\
\qquad [D + \underline{\ }_1]x_2(t), \\
\dot{x}_3(t) = Dx_2(t) + \underline{\ }_1 x_3(t),
\end{cases}
$$

where the predator population is divided into mature and immature predator, and $x_1(t)$ denotes the density of prey at time t, $x_2(t)$ and $x_3(t)$ represents densities of the immature individual predator and mature individual predator at time t respectively. The material biological meaning can be seen in Reference [2].

However, the model ignores the functional response and harvesting predator. Recently, Song and Chen[8] also investigated a stage-structured population model with two life stages, immature and mature, with harvesting mature population and stocking immature population. Dong et al[9] and Jiao[10] investigated the extinction and permanence of the predator-prey system with stocking of prey and harvesting of predator impulsively. There are papers[10~14] studied the model with time delay and functional response.

Motivated by the recent research mentioned above, and to make the model is more close to reality, certain research has been carried out in the local sea area (Guangxi Beibu Gulf), seeing fishermen using fish and shrimp for Mari-culture, and we consider the following model:

$$\begin{cases} \dot{x}_1(t) = s(t)x_2(t) - r(t)x_1(t) - \\ \quad a_{11}(t)x_1(t) - b_{12}x_1^2(t), \\ \dot{x}_2(t) = r(t)x_1(t) - a_{21}(t)x_2(t) - \\ \quad b_{22}x_2^2(t) - \dfrac{T(t)x_2^2(t)y(t)}{1+ U(t)x_2^2(t)}, \\ \dot{y}_1(t) = - a_{31}(t)y(t) - b_{32}y^2(t) + d(t)\cdot \\ \quad \dfrac{T(t)x_2^2(t)(t - f(t,y(t))y(t)}{1+ U(t)x_2^2(t - f(t,y(t))} - \\ \quad E(t)y(t), \end{cases}$$

(0.1)

where x, y is prey species and predator species respectively. $y(t)$ denotes the density of predator at time t, $x_1(t)$ and $x_2(t)$ represents densities of the immature individual prey and mature individual prey at time t respectively. $a_{i1}(t) > 0 (i = 1, 2, 3)$ is the death rate, and $b_{i2}(t) > 0 (i = 1, 2, 3)$ is the coefficient intra-specific competition of the immature individual prey and mature individual prey and predator respectively; $\frac{x_2^2(t - f)y(t)}{1+ U(t)x_2^2(t - f)}$ is the functions with type III functional response, $T(t)$ is the capture rate of predator, $d(t) > 0$ is the rate of conversion of nutrients into the reproduction rate of predator; $1 > E(t) > 0$ is the effect of continuous harvesting predator.

All of the above variable coefficients ($\in C(R, R^+)$, $R^+ = (0, +\infty)$) are positive periodic continuous functions with period $k > 0$. The purpose of this paper is to study the existence of positive periodic solution of system (0.1) by using continuation theorem of coincidence degree theory which was proposed in Reference [15].

1 Lemma

Let X and Y be real Banach spaces, L: Dom$L \subset X \to Y$ a Fredholm mapping of index zero and P: $X \to X$, Q: $Y \to Y$ continuous projectors such that Im P = Ker$L$, Ker$Q$ = ImL and X = Ker$L \oplus$ Ker$P$, Y = Im$L \oplus$ ImQ. Denoted the restriction of L by L_P in Dom$L \cap$ Ker$P$, K_P: Im$L \to$ Ker$P \cap$ Dom$L$ the inverse to L_P and J: Im$Q \to$ Ker$L$ an isomorphism of ImQ onto Ker$L$.

Lemma 1.1[15] Let $K \subset X$ be an open bounded set and N: $X \to Y$ be a continuous operator which is L-compact on $\overline{K}$ (i.e. QN: $\overline{K} \to Y$ and $K_P(I - Q)N$: $\overline{K} \to Y$ are compact). Assume (a) for each $\lambda \in (0, 1)$, $x \in \partial K \cap$ Dom$L$, $Lx \neq \lambda Nx$; (b) for each $x \in \partial K \cap$ Ker$L$, $QNx \neq 0$; (c) deg$\{JQN, K \cap$ Ker$L, 0\} \neq 0$. Then $Lx = Nx$ has at least one periodic solution in $\overline{K} \cap$ Dom$L$.

2 Main result

In the following, we use the notations:

$$\bar{f} = \frac{1}{k}\int_0^k f(t)\mathrm{d}t, f^L = \min_{t\in[0,k]}|f(t)|, f^M = \min_{t\in[0,k]}|f(t)|,$$

where f is a continuous k-periodic function.

Theorem 2.1 Assume the following conditions

(H1) $f(t, y(t))$ being continuous and k-periodic with respect to t and due to gestation of y,

(H2) $(k - r - a_{11})^L > 0$,

(H3) $[(r - a_{21})^L]^2 > 4b_{22}^M[(\frac{T}{U})^M]^2 \cdot \frac{d^M}{b_{32}^L}$,

(H4) $\left(\frac{dT}{1+ U} - a_{31} - E\right)^L > 0$

are satisfied. Then system (0.1) has at least one positive k-periodic solution.

Proof Let $x_i(t) = e^{u_i(t)}$, $i = 1, 2$, $y(t) = e^{u_3(t)}$, then system (0.1) becomes

$$\begin{cases} \dot{u}_1(t) = s(t)e^{u_2(t) - u_1(t)} - r(t) - a_{11}(t) - \\ \quad b_{12}(t)e^{u_1(t)}, \\ \dot{u}_2(t) = r(t)e^{u_1(t) - u_2(t)} - a_{21}(t) - b_{22}(t)e^{u_2(t)} - \\ \quad \dfrac{T(t)e^{u_2(t)}e^{u_3(t)}}{1+ U(t)e^{2u_2(t)}}, \\ \dot{u}_3(t) = - a_{31}(t) - b_{32}(t)e^{u_3(t)} + \\ \quad d(t)\dfrac{T(t)e^{2u_2(t - f(t, e^{u_3(t)}))}}{1+ U(t)e^{2u_2(t - f(t, e^{u_3(t)}))}} - \\ \quad E(t). \end{cases}$$

(2.1)

It is easy to see that if system (2.1) has one k-periodic solution $(u_1^*(t), u_2^*(t), u_3^*(t))^T$, then $(e^{u_1^*(t)}, e^{u_2^*(t)}, e^{u_3^*(t)})^T$ is a positive -periodic solution of system (0.1). Then, the next work is to prove that system (2.1) has one k-periodic solution.

In order to apply the continuation theorem of coincidence degree theory to establish the existence of k-periodic solution of system (2.1), we take $X = Y = \{u = (u_1(t), u_2(t), u_3(t))^T \in C(R, R^3)$: $u_i(t + k) =$

$u_i(t), i = 1, 2, 3\}$ and $\|(u_1(t), u_2(t), u_3(t))^T\| = \sum_{i=1}^{3} \max_{t\in[0,\omega]} |u_i(t)|$, here, $|\cdot|$ denotes the Euclidean norm. With this norm $\|\cdot\|$, X and Y are Banach spaces. Set L: $\mathrm{Dom}L \cap X$, $L(u_1(t), u_2(t), u_3(t))^T = (\dot{u}_1(t), \dot{u}_2(t), \dot{u}_3(t))^T$, where $\mathrm{Dom}L = \{u(t) = (u_1(t), u_2(t), u_3(t))^T \in C^1(R, R^3)\}$, and N: $X \to X$, $Lu = Nu$. Define two projectors P and Q as $Pu = Qu = \frac{1}{\omega}\int_0^{\omega} u\mathrm{d}t$, $u \in X$.

Clearly, $\mathrm{Ker}L = R^3$, $\mathrm{Im}L = \{(u_1(t), u_2(t), u_3(t))^T \in X: \int_0^{\omega} u_i(t)\mathrm{d}t = 0, i = 1, 2, 3\}$ is closed in X and $\dim \mathrm{Ker}L = \mathrm{codim}\ \mathrm{Im}L = 3$, therefore, L is a Fredholm mapping of index zero. Through computation we find that the inverse K_P of L_P has the form K_P: $\mathrm{Im}L \to \mathrm{Dom}L \cap \mathrm{Ker}P$, $K_P u = \int_0^t u(s)\mathrm{d}s - \frac{1}{\omega}\int_0^{\omega}\int_0^t u(s)\mathrm{d}s\mathrm{d}t$. Moreover, we have

$$K_P(I - Q)Nu = K_PNu - K_PQNu =$$

$$\int_0^t \dot{u}(s)\mathrm{d}s - \frac{1}{\omega}\int_0^{\omega}\int_0^{\eta} \dot{u}(s)\mathrm{d}s\mathrm{d}\eta - [\frac{t}{\omega}\int_0^{\omega} \dot{u}(s)\mathrm{d}s - \frac{1}{\omega^2}\int_0^{\omega}(\eta\int_0^{\omega} \dot{u}(s)\mathrm{d}s)\mathrm{d}\eta] = \int_0^t \dot{u}(s)\mathrm{d}s - \frac{1}{\omega}\int_0^{\omega}\int_0^{\eta} \dot{u}(s)\mathrm{d}s\mathrm{d}\eta - (\frac{t}{\omega} - \frac{1}{2})\int_0^{\omega} \dot{u}(s)\mathrm{d}s.$$

We can show that QN and $K_P(I - Q)N$ are continuous by Lebesgue convergence theorem and that $QN(\bar{\Omega})$, $K_P(I - Q)N(\bar{\Omega})$ are relatively compact for any open bounded subset Ω by Arzela-Ascoli theorem. Therefore, N is L-compact on $\bar{\Omega}$ for any open bounded subset $\Omega \in X$. Corresponding to the operator equation $Lu = \lambda Nu, \lambda \in (0, 1)$, we have

$$\begin{cases} \dot{u}_1(t) = \lambda[s(t)e^{u_2(t) - u_1(t)} - r(t) - a_{11}(t) - b_{12}(t)e^{u_1(t)}], \\ \dot{u}_2(t) = \lambda[r(t)e^{u_1(t) - u_2(t)} - a_{21}(t) - b_{22}(t)e^{u_2(t)} - \dfrac{\alpha(t)e^{u_2(t)}e^{u_3(t)}}{1 + \beta(t)e^{2u_2(t)}}], \\ \dot{u}_3(t) = \lambda[- a_{31}(t) - b_{32}(t)e^{u_3(t)} + d(t)\dfrac{\alpha(t)e^{2u_2(t - \tau(t, e^{u_3(t)}))}}{1 + \beta(t)e^{2u_2(t - \tau(t, e^{u_3(t)}))}} - E(t)]. \end{cases} \quad (2.2)$$

Choose $\xi_i, \eta_i \in [0, \omega]$ such that $u_i(\xi_i) = \max_{t\in[0,\omega]} u_i(t)$, $u_i(\eta_i) = \min_{t\in[0,\omega]} u_i(t)$, $i = 1, 2, 3$. Then it is clear that $\dot{u}_i(\xi_i) = \dot{u}_i(\eta_i) = 0$, $i = 1, 2, 3$. From this and system (2.2), we obtain

$$s(\xi_1)e^{u_2(\xi_1) - u_1(\xi_1)} - r(\xi_1) - a_{11}(\xi_1) - b_{12}(\xi_1)e^{u_1(\xi_1)} = 0; r(\xi_2)e^{u_1(\xi_2) - u_2(\xi_2)} - a_{21}(\xi_2) - b_{22}(\xi_2)e^{u_2(\xi_2)} - \frac{\alpha(\xi_2)e^{u_2(\xi_2)}e^{u_3(\xi_2)}}{1 + \beta(\xi_2)e^{2u_2(\xi_2)}} = 0; - a_{31}(\xi_3) - b_{32}(\xi_3)e^{u_3(\xi_3)} + d(\xi_3)\frac{\alpha(\xi_3)e^{2u_2(\xi_3 - \tau(\xi_3, e^{u_3(\xi_3)}))}}{1 + \beta(\xi_3)e^{2u_2(\xi_3 - \tau(\xi_3, e^{u_3(\xi_3)}))}} - E(\xi_3) = 0, \quad (2.3)$$

and

$$s(\eta_1)e^{u_2(\eta_1) - u_1(\eta_1)} - r(\eta_1) - a_{11}(\eta_1) - b_{12}(\eta_1)e^{u_1(\eta_1)} = 0; r(\eta_2)e^{u_1(\eta_2) - u_2(\eta_2)} - a_{21}(\eta_2) - b_{22}(\eta_2)e^{u_2(\eta_2)} - \frac{\alpha(\eta_2)e^{u_2(\eta_2)}e^{u_3(\eta_2)}}{1 + \beta(\eta_2)e^{2u_2(\eta_2)}} = 0; - a_{31}(\eta_3) - b_{32}(\eta_3)e^{u_3(\eta_3)} + d(\eta_3)\frac{\alpha(\eta_3)e^{2u_2(\eta_3 - \tau(\eta_3, e^{u_3(\eta_3)}))}}{1 + \beta(\eta_3)e^{2u_2(\eta_3 - \tau(\eta_3, e^{u_3(\eta_3)}))}} - E(\eta_3) = 0. \quad (2.4)$$

If $u_1(\xi_1) \geqslant u_2(\xi_2)$, then $u_1(\xi_1) \geqslant u_2(\xi_2) \geqslant u_2(\xi_1)$. From the first formula of (2.3), we have

$$b_{12}(\xi_1)e^{u_1(\xi_1)} \leqslant s(\xi_1), e^{u_1(\xi_1)} \leqslant \frac{s^M}{b_{12}^L} := \rho_1. \quad (2.5)$$

If $u_1(t_1) < u_2(t_2)$, then $u_1(t_2) \leqslant u_1(t_1) < u_2(t_2)$. By the second formula of (2.3), we get

$$b_{22}(\xi_2)e^{u_2(\xi_2)} < r(\xi_2) \Rightarrow e^{u_2(\xi_2)} < \frac{r^M}{b_{22}^L} := \rho_2. \quad (2.6)$$

Set $\rho = \max\{\rho_1, \rho_2\}$, by fromula (2.5) and fromula (2.6), for $\forall t \in [0, \omega]$, we have

$$e^{u_1(\xi_1)} < \rho, e^{u_2(\xi_2)} < \rho. \quad (2.7)$$

From the last formula of (2.3), we have $b_{32}(\xi_3)e^{u_3(\xi_3)} \leqslant d(\xi_3)\frac{\alpha(\xi_3)}{\beta(\xi_3)}$, so

$$e^{u_3(\xi_3)} < (\frac{d\alpha}{b_{32}\beta})^M := \rho_3. \quad (2.8)$$

If $u_1(\eta_1) \leqslant u_2(\eta_2)$, then $u_1(\eta_1) \leqslant u_2(\eta_2) \leqslant u_2(\eta_1)$. By the first formula of (2.4), we get $b_{12}(\eta_1)e^{u_1(\eta_1)} = s(\eta_1)e^{u_2(\eta_1) - u_1(\eta_1)} - r(\eta_1) - a_{11}(\eta_1)$, so

$$e^{u_1(\eta_1)} > \frac{1}{b_{12}^M}[s - r - a_{11}]^L := \sigma_1. \quad (2.9)$$

If $u_1(\eta_1) > u_2(\eta_2)$, then $u_1(\eta_2) \geqslant u_1(\eta_1) > u_2(\eta_2)$. By the second formula of (2.4), we obtain

$$b_{22}(\eta_2)e^{u_2(\eta_2)} = r(\eta_2)e^{u_1(\eta_2) - u_2(\eta_2)} - a_{21}(\eta_2) - \frac{\alpha(\eta_2)e^{u_2(\eta_2)}e^{u_3(\eta_2)}}{1 + \beta(\eta_2)e^{2u_2(\eta_2)}} > r(\eta_2) - a_{21}(\eta_2) - \frac{\alpha(\eta_2)e^{u_3(\eta_2)}}{\beta(\eta_2)e^{u_2(\eta_2)}} \Rightarrow e^{u_2(\eta_2)} > [(r - a_{21})^L + \sqrt{[(r - a_{21})^L]^2 - 4b_{22}^M[(\frac{\alpha}{\beta})^M]^2 \cdot \frac{d^M}{b_{32}^L}}]/(2b_{22}^M) := \sigma_2. \quad (2.10)$$

Set $\sigma = \min\{\sigma_1, \sigma_2\}$, for $\forall t \in [0, \omega]$, we have

$$e^{u_1(\eta_1)} > \sigma, e^{u_2(\eta_2)} > \sigma. \quad (2.11)$$

From the last formula of (2.4) we have

$$b_{32}(Z_3)e^{u_3(Z_3)} = -a_{31}(Z_3) + d(Z_3)\cdot$$

$$\frac{\mathrm{T}(Z_3)e^{2u_2(Z_3 - \mathrm{f}(Z_3, e^{u_3(Z_3)}))}}{1 + \mathrm{U}(Z_3)e^{2u_2(Z_3 - \mathrm{f}(Z_3, e^{u_3(Z_3)}))}} - E(Z_3) > -a_{31}(Z_3) +$$

$$d(Z_3)\frac{\mathrm{T}(Z_3)}{1 + \mathrm{U}(Z_3)} - E(Z_3),$$

and then

$$e^{u_3(Z_3)} > \frac{1}{b_{32}^M}\left[-a_{31} + \frac{d\mathrm{T}}{1 + \mathrm{U}} - E\right]^L := \mathrm{W}_3. \quad (2.12)$$

Therefore, from formula (2.11), formula (2.12), it follows that there exist two positive constants W, W_3, such that

$$e^{u_i(t)} > \mathrm{W}(i = 1, 2), e^{u_3(t)} > \mathrm{W}_3. \quad (2.13)$$

From formula (2.7), formula (2.8) and formula (2.13), we obtain $|u_i(t)| < \max\{|\ln d|, |\ln d_3|, |\ln \mathrm{W}|, |\ln \mathrm{W}_3|\} =: R_i, i = 1, 2, 3$. Obviously, $R_i (i = 1, 2, 3)$ are independent of λ.

Using the integral mean valued theorem, it follows that there exist some points $\mathrm{Y}_i \in [0, \mathrm{k}] (i = 2, 3)$ such that when $(u_1, u_2, u_3)^{\mathrm{T}}$ is a constant vector, we get

$$QN\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} =$$

$$\begin{bmatrix} \bar{s}e^{u_2 - u_1} - \bar{r} - \bar{a}_{11} - \bar{b}_{12}e^{u_1} \\ \bar{r}e^{u_1 - u_2} - \bar{a}_{21} - \bar{b}_{22}e^{u_2} - \frac{\overline{\mathrm{T}}e^{u_2}e^{u_3}}{1 + \mathrm{U}(\mathrm{Y}_2)e^{2u_2}} \\ -\overline{(a_{31} + E)} - \bar{b}_{32}e^{u_3} + \frac{\overline{d\mathrm{T}}e^{2u_2}}{1 + \mathrm{U}(\mathrm{Y}_3)e^{2u_2}} \end{bmatrix}. \quad (2.14)$$

Denote $M = \sum_{i=1}^{3} R_i + R_0$, R_0 is taken sufficient large such that each solution $(\mathrm{T}^*, \mathrm{U}^*, \mathrm{V}^*)^{\mathrm{T}}$ of the system:

$$\begin{cases} \bar{s}e^{\mathrm{U} - \mathrm{T}} - \bar{r} - \bar{a}_{11} - \bar{b}_{12}e^{\mathrm{T}} = 0, \\ \bar{r}e^{\mathrm{T} - \mathrm{U}} - \bar{a}_{21} - \bar{b}_{22}e^{\mathrm{U}} - \frac{\overline{\mathrm{T}}e^{\mathrm{U}}e^{\mathrm{V}}}{1 + \mathrm{U}(\mathrm{Y}_2)e^{2\mathrm{U}}} = 0, \\ -\overline{(a_{31} + E)} - \bar{b}_{32}e^{\mathrm{V}} + \frac{\overline{d\mathrm{T}}e^{2\mathrm{U}}}{1 + \mathrm{U}(\mathrm{Y}_3)e^{2\mathrm{U}}} = 0, \end{cases} \quad (2.15)$$

satisfies $\|(\mathrm{T}^*, \mathrm{U}^*, \mathrm{V}^*)^{\mathrm{T}}\| = |\mathrm{T}^*| + |\mathrm{U}^*| + |\mathrm{V}^*| < M$. We will prove that system (2.15) has a solution or a number of solutions. Now we take $\mathrm{K} = \{u = (u_1, u_2, u_3)^{\mathrm{T}} \in X: \|u\| < M\}$. This satisfies condition (a) in Lemma 1.1. When $(u_1, u_2, u_3)^{\mathrm{T}} \in \partial\mathrm{K} \cap \mathrm{Ker}L = \partial\mathrm{K} \cap R^3$, $(u_1, u_2, u_3)^{\mathrm{T}}$ is a constant vector in R^3 with $\sum_{i=1}^{3}|u_i| = M$. If system (2.15) has a solution or a number of solutions, then $QN(u_1, u_2, u_3)^{\mathrm{T}} \neq (0, 0, 0)^{\mathrm{T}}$. If there is no solution of system (2.15), then naturally

$$QN\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (2.16)$$

This proves that condition (b) in Lemma 1.1 is satisfied.

Finally we will show that condition (c) of Lemma 1.1 is satisfied. So, we define $\mathrm{Q}: \mathrm{Dom}L \times [0, 1] \to X$, by

$$\mathrm{Q}(u_1, u_2, u_3, \_) = \begin{bmatrix} \bar{s}e^{u_2 - u_1} - \bar{r} \\ \bar{r}e^{u_1 - u_2} - \bar{a}_{21} \\ -\bar{b}_{32}e^{u_3} + \frac{\overline{d\mathrm{T}}e^{2u_2}}{1 + \mathrm{U}(\mathrm{Y}_3)e^{2u_2}} \end{bmatrix} +$$

$$\_\begin{bmatrix} -\bar{a}_{11} - \bar{b}_{12}e^{u_1} \\ -\bar{b}_{22}e^{u_2} - \frac{\overline{\mathrm{T}}e^{u_2}e^{u_3}}{1 + \mathrm{U}(\mathrm{Y}_2)e^{2u_2}} \\ -\overline{(a_{31} + E)} \end{bmatrix}, \quad (2.17)$$

where $\_ \in [0, 1]$ is a parameter.

When $(u_1, u_2, u_3)^{\mathrm{T}} \in \partial\mathrm{K} \cap \mathrm{Ker}L = \partial\mathrm{K} \cap R^3$, $(u_1, u_2, u_3)^{\mathrm{T}}$ is a constant vector in R^3 with $\sum_{i=1}^{3}|u_i| = M$. We will show that when $(u_1, u_2, u_3)^{\mathrm{T}} \in \partial\mathrm{K} \cap \mathrm{Ker}L$, $\mathrm{Q}(u_1, u_2, u_3, \_) \neq (0, 0, 0)^{\mathrm{T}}$. Otherwise, constant vector $(u_1, u_2, u_3)^{\mathrm{T}}$ with $\sum_{i=1}^{3}|u_i| = M$ satisfies $\mathrm{Q}(u_1, u_2, u_3, \_) = (0, 0, 0)^{\mathrm{T}}$, then from

$$\bar{s}e^{u_2 - u_1} - \bar{r} - \_(\bar{a}_{11} + \bar{b}_{12}e^{u_1}) = 0; \bar{r}e^{u_1 - u_2} - \bar{a}_{21} - \_(\bar{b}_{22}e^{u_2} + \frac{\overline{\mathrm{T}}e^{u_2}e^{u_3}}{1 + \mathrm{U}(\mathrm{Y}_2)e^{2u_2}}) = 0; -\bar{b}_{32}e^{u_3} + \frac{\overline{d\mathrm{T}}e^{2u_2}}{1 + \mathrm{U}(\mathrm{Y}_3)e^{2u_2}} - \_\overline{(a_{31} + E)} = 0, \quad (2.18)$$

by above-mentioned argument formula (2.7), formula (2.8) and formula (2.13), magnifying $\bar{f}$ into f^M and reducing $\bar{f}$ into f^L, and magnifying $\_$ into 1 and reducing $\_$ into 0, we get $|u_i| < \max\{|\ln d|, |\ln d_3|, |\ln \mathrm{W}|, |\ln \mathrm{W}_3|\}, i = 1, 2, 3$. Then $\sum_{i=1}^{3}|u_i| < \sum_{i=1}^{3}R_i < M$, which contradicts the fact that constant vector $(u_1, u_2, u_3)^{\mathrm{T}}$ satisfies $\sum_{i=1}^{3}|u_i| = M$. Therefore, $\mathrm{Q}(u_1, u_2, u_3, \_) \neq (0, 0, 0)^{\mathrm{T}}$, for $(u_1, u_2, u_3)^{\mathrm{T}} \in \partial\mathrm{K} \cap \mathrm{Ker}L$. Using the property of topological degree and take $J = I: \mathrm{Im}L \to \mathrm{Ker}L$, we have

$\deg\{JQN(u_1,u_2,u_3,\_)^T, K \cap \mathrm{Ker}L, (0,0,0)^T\} = \deg\{Q(u_1,u_2,u_3,1), K \cap \mathrm{Ker}L, (0,0,0)^T\} = \deg\{Q(u_1,u_2,u_3,0), K \cap \mathrm{Ker}L, (0,0,0)^T\} = -\mathrm{sgn}(\bar{s}\,\bar{T}v\bar{b}_{32}e^{-u_1-u_2+u_3}) = -1 \neq 0$. This completes the proof of condition(c) of Lemma 1. 1.

By now we know that K satisfies all the requirements of Lemma 1. 1. So, system (0. 1) has at least one positive k -periodic solution.

From Theorem 2. 1, it is clear that the excess harvesting causes the extinction of the predator population, and breaches the sustainable development of biological resources. That is, the behavior of reasonable harvesting can bring the permanence of the exploitative predator-prey system. However, there is an interesting problem: how to do impulsive stocking on prey and for predator optimal harvesting. We will continue to study such problems in the future.

References

[1] Chen L S, Wang D D, Yang Q C. Stage-structured population dynamics model [J]. Journal of Beihua University, 2000, 1(3): 185-197.

[2] Wang W D, Chen L S. A predator-prey system with stage structure for predator[J]. Computer Math Applics, 1997, 33(8): 83-91.

[3] Liu S Q, Chen L S, Luo G L, et al. Asymptotic behaviors of competitive Lotka-Volterra system with stage structure[J]. J Math Anal Appl, 2002, 271: 124-138.

[4] Zeng W W, Cui J A. Control of population size and stability through diffusion: a stage-structure model [J]. Journal of Biomathematics, 2001, 16(3): 275-280.

[5] Liu Q. Persistence and periodic solution of mixed model with Holling III functional response and stage-structure for prey[J]. Journal of Guangxi University, 2005, 30(1): 85-88.

[6] Liu K Y, Chen L S. On a periodic time-dependent model of population dynamics with stage structure and impulsive effects [J]. Discrete Dynamics in Nature and Society, Vol 2008, Article ID 706154, 13 Pages, 2008.

[7] Zeng G Z, Sun L H. Permanence and existence of periodic solution of nonautonomous predator-prey system with stage structure and delay [J]. Journal of Biomathematics, 2005, 20(2): 311-320.

[8] Song X Y, Chen L S. Optimal harvesting policy and stability for single-species growth model with stage-structured [J]. Journal of System Sciences and Complex, 2002, 15(2): 194-201.

[9] Dong L Z, Chen L S, Sun L H. Extinction and persistence of the predator-prey system with stocking of prey and harvesting of predator impulsively [J]. Math Methods Appl Sci, 2006, 29(4): 415-425.

[10] Jiao J J, Zhang W Z. A mutual interference stage-structured predator-prey model with time delay and impulsive perturbations on predators [J]. Sys Sci & Math Scis, 2008, 28(7): 791-801.

[11] Zhang Z Q, Wang Z C. The existence of periodic solution of a two-patches predator-prey dispersion delay models with functional response[J]. J Korean Math Soc 40, 2003, 5: 869-881.

[12] Li B W, Zeng X W. Existence of positive solution for a two-patches competition system with diffusion and time delay and functional response[J]. Appl Math J Chinese Univ Ser B, 2003, 18(1): 1-8.

[13] Liu Q. Almost periodic solution of a diffusive mixed system with time delay and type III functional response [J]. Discrete Dynamics in Nature and Society, Vol 2008, Article ID 706154, 13 Pages, 2008.

[14] Liu Q. Periodic solution of diffusive predator-prey system time delay and type III functional response[J]. International Journal of Pure and Applied Mathematics, 2008, 47(1): 53-64.

[15] Gains R E, Mawhins J L. Coincidence degree and nonlinear differential equations [M]. Berlin: Springer, 1977.

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