DOI: 10.13656/j.cnki.gxkx.2009.01.003
广西科学 Guangxi Sciences 2009, 16(1): 7~ 8

Some New Nonhamilton Graphs*
一类新的非哈密顿图

TANG Gan-wu[1], TANG Gao-hua[2], WANG Min[3]
唐干武[1],唐高华[2],王 敏[3]

(1. Department of Mathematics and Computer Science, Guilin Normal College, Guilin, Guangxi, 541001, China; 2. Department of Mathematics and Computer Science, Guangxi Teachers Education University, Nanning, Guangxi, 530023, China; 3. Department of Mathematics and Information Science, Yantai University, Yantai, Shandong, 264005, China.)
(1.桂林师范高等专科学校数学与计算机科学系,广西桂林 541001; 2.广西师范学院数学与计算机科学系,广西南宁 530023; 3.烟台大学数学与信息科学系,山东烟台 264005)

Abstract A class of famous nonhamilton-graphs $K_m \vee (\overline{K_m} + K_{n-2m})$ $(1 \leqslant m \leqslant \frac{n}{2})$ have been extened to $K_s \vee (\sum_{i=1}^{s+1} K_{m_i})$ $(\sum_{i=1}^{s+1} m_i = n-s, n \geqslant 3, 1 \leqslant s \leqslant \frac{n-1}{2})$ and their simple properties are discussed.

Key words nonhamilton-graphs, $C_{n,m}$-graphs, $R_{s,n}$-graphs

摘要: 把非哈密顿图 $K_m \vee (\overline{K_m} + K_{n-2m})$ $(1 \leqslant m \leqslant \frac{n}{2})$扩充为 $K_s \vee (\sum_{i=1}^{s+1} K_{m_i})$ $(\sum_{i=1}^{s+1} m_i = n-s, n \geqslant 3, 1 \leqslant s \leqslant \frac{n-1}{2})$,并讨论此类图的简单性质.

关键词: 非哈密顿图 $C_{n,m}$图 $R_{s,n}$图
中图法分类号: O157.5 文献标识码: A 文章编号: 1005-9164(2009)01-0007-02

Bondy. J. A and Murty. U. S. R[1] have defined a class of graphs $K_m \vee (\overline{K_m} + K_{n-2m})$ which is denoted by $C_{n,m}$ $(1 \leqslant m \leqslant \frac{n}{2})$, and have proved that the $C_{n,m}$ is a class of nonhamilton graphs. In this paper, we denote $K_s \vee (\sum_{i=1}^{s+1} K_{m_i})$ graph as $R_{s,n}$ and if $m_1, m_2, \cdots, m_{s+1}$ are given we denote as $R_{s,n(m_1,m_2,\cdots,m_{s+1})}$, where $\sum_{i=1}^{s+1} m_i = n-s, n \geqslant 3, 1 \leqslant s \leqslant \frac{n-1}{2}$.

1 Preliminaries

Suppose G is a simple undirected graphs, $V(G)$ and $E(G)$ denote vertex set and edge set in graph G respectively. We say G is a $(p, p-k)$-graph if $|E(G)| = |V(G)| - k$, where k is integer. A m-partite graph is that whose vertex set can be partitioned into m subsets and no edge has both ends in any one subset. A simple graph is said to be a complete m-partite graph if it satisfies that each vertex is joined to every vertex that is not in the same subset. The number of edges in graph G is denoted by $X(G)$, i.e., $X(G) = |E(G)|$. Let $k(G)$ denote the number of components in graph G, $\overline{G}$ denote the complement of graph G, K_n denote the complete graph with n vertices. The union of $G_1, G_2, \cdots, G_k$, is denoted by $G_1 \cup G_2 \cup \cdots \cup G_k$ and when $G_1, G_2, \cdots, G_k$ are pairwise disjoint, $G_1 \cup G_2 \cup \cdots \cup G_k$ is denoted by $G_1 + G_2 + \cdots + G_k$ or $\sum_{i=1}^{k} G_i$. We say the graph is the join of pairwise disjoint graphs $G_1, G_2, \cdots, G_k$, which

收稿日期: 2008-05-05
作者简介: 唐干武(1962-),男,副教授,主要从事图论及概率统计研究工作。
* Supported by the Foundation of Guangxi Natural Sciences (0221029) and the Foundation of Guangxi Educational Department (200807MS032).

is obtained by joining each vertex of G_i to every vertex of $G_1+G_2+\cdots+G_{i-1}+G_{i+1}+\cdots+G_k$ for any $i\in\{1,2,\cdots,k\}$ in $G_1+G_2+\cdots+G_k$, and denote as $G_1\vee G_2\vee\cdots\vee G_k$. A Hamilton cycle of graph G is a cycle that contains every vertex of graph G. A graph is a Hamilton graph if it contains a Hamilton cycle. A graph is a nonhamilton graph if it doesn't contain any Hamilton cycle. Other notations and terminologies not defined here can be found in reference[1].

Lemma 1.1[1] Suppose G is a Hamilton graph, then $k(G-D)\leqslant |D|$ for every nonempty proper subset D of $V(G)$.

2 Main results

In the following, we prove that $R_{s,n}$-graphs are nonhamilton graphs and discuss its some properties. Furthermore, $R_{s,n}$ is a bigger class than $C_{n,m}$.

Lemma 2.1 Let G be a complete m-partite graph with n vertices. We deonte the complete m-partite graph with n vertices as $T_{m,n}$ which satisfies that the number of vertices of a part is more than one, and the rest is only one. Then we get $\varepsilon(G)\geqslant\varepsilon(T_{m,n})$, with equality only if G is isomorphic to $T_{m,n}$.

Proof Suppose G is a complete m-partite graph with n vertices, and the numbers of vertices of each parts are $n_1,n_2,\cdots,n_m$ respectively. Without loss of generality, suppose $n_1\leqslant n_2\leqslant\cdots\leqslant n_m$, if G isn't isomorphic to $T_{m,n}$, then there exists $i\in\{1,2,\cdots,m-1\}$ such that $n_i>1$. Let's consider complete m-partite graph with n vertices G_1 which the numbers of vertices of each parts are $n_1,n_2,\cdots,n_i-1,\cdots,n_m+1$, respectively, then

$$\varepsilon(G_1)=\frac{1}{2}\sum_{k=1,k\neq i}^{m-1}(n-n_k)n_k+\frac{1}{2}(n-n_i+1)(n_i-1)+\frac{1}{2}(n-n_m-1)(n_m+1)=\frac{1}{2}\sum_{k=1}^{m}(n-n_k)n_k-(n_m-n_i+1)<\varepsilon(G)=\frac{1}{2}\sum_{k=1}^{m}(n-n_k)n_k.$$

If G_1 is isomporphic to $T_{m,n}$, then Lemma 2.1 holds. Otherwise, we continue above process until $n_j=1$ for any $j\in\{1,2,\cdots,m-1\}$, then we obtain the graph is $T_{m,n}$. Noting that the edges of graph which we obtain by above process is reducing gradually to $\varepsilon(T_{m,n})$. So we complete the proof of Lemma 2.1.

Theorem 2.1 $C_{n,m}$ is a subset of $R_{s,m}$.

Theorem 2.2 $R_{s,m}$ is a class of nonhamilton graphs.

Theorem 2.3 The inequality $\varepsilon(\overline{R_{s,n}})\geqslant\frac{1}{2}s(2n-3s-1)$ is true.

Theorem 2.4 Let $\overline{R_{s,n}}$ be a (p,q)-graph, where q is less than or equal to $p+1$, then s is less than or equal to 4. Further, when s is unequal to 1, then n is bigger than or equal to 5 and is less than or equal to 9.

Proof of Theorem 2.1 Because of the denotations of $C_{n,m}$ and $R_{s,n}$, therefore $C_{n,m}$ is a special case of $R_{s,n}$, i.e. $C_{n,m}=R_{m,n(1,1,\cdots,1,n-2m)}=K_m\vee(\sum_{i=1}^{m}K_1+K_{n-2m})$. So $C_{n,m}$ is a subset of $R_{s,n}$. We complete the proof of Theorem 2.1.

Proof of Theorem 2.2 We denote the subset K_s which contains s vertices of $V(R_{s,n})$ as D, then we have $k(R_{s,n}-D)=s+1>|D|$. By Lemma 1.1, $R_{s,n}$ is a nonhmailton graph. So we complete the proof of Theorem 2.2.

Proof of Theorem 2.3 Since $\bigvee_{i=1}^{s+1}\overline{K_{m_i}}$ is a complete $(s+1)$-partite graph with $n-s$ vertices and $\overline{R_{s,n}}=\overline{K_s}+(\bigvee_{i=1}^{s+1}\overline{K_{m_i}})$, then by Lemma 2.1, it is easy to see that $\varepsilon(\overline{R_{s,n}})=\varepsilon(\overline{K_s}+(\bigvee_{i=1}^{s+1}\overline{K_{m_i}}))=\varepsilon(\bigvee_{i=1}^{s+1}\overline{K_{m_i}})\geqslant\varepsilon(T_{s+1,n-s})=\frac{1}{2}s(2n-3s-1)$. So we get the result.

Proof of Theorem 2.4 Because of the definition of $\overline{R_{s,n}}$ and Theorem 2.3 and $q\leqslant p+1$, we have

$$\begin{cases}1\leqslant s\leqslant\dfrac{n-1}{2},\\[2mm] 0\leqslant\dfrac{1}{2}s(2n-3s-1)\leqslant n+1.\end{cases}$$

It is easy to see that Theorem 2.4 holds.

References

[1] Bondy J A, Murty U S R. Graph theory with applications[M]. New York: Macmillan Press, 1976.

(责任编辑: 尹 闯)