DOI: 10.13656/j.cnki.gxkx.2008.03.013
广西科学 Guangxi Sciences 2008, 15(3): 247~ 249

一类六对称五次多项式微分系统的小振幅极限环分支*

Small Limit Cycles Bifurcating from Fine Focus Points in Quintic Order Z_6-equivariant Polynomial System

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摘要: 研究一类六对称五次多项式微分系统的小振幅极限环分支问题,给出该系统奇点量的递推公式和系统的焦点量,并推导出这类六对称五次多项式系统在6个细焦点可以分支出12个小振幅极限环.
关键词: 微分系统 细焦点 奇点量 极限环分支
中图法分类号: O175.12 文献标识码: A 文章编号: 1005-9164(2008)03-0247-03
Abstract In this work, we study bifurcation of small limit cycles for a class of quintic order Z_6-equivariant polynomial system, and the recursive formula of computing singular point values and the focus point values of this system are getten. It shows that this system allows the appearance of twelve limit cycles in six fine focus points.
Key words differential system, fine focus point, singular point value, bifurcation of limit cycles

多项式系统的极限环问题与 Hilbert第16问题[1]的后半部分相关,是微分方程定性理论与分支理论的热点问题.近年来许多学者对对称多项式系统的极限环进行了研究[1~8].文献[2,3]证明两类二对称三次多项式微分系统可分支出12个小振幅极限环.文献[4]证明一类三对称四次多项式微分系统可分支出16个小振幅极限环.文献[5]证明一类经过四对称五次扰动的三次多项式微分系统在焦点分支出16个极限环.文献[6]证明一类五对称五次系统可分支出25个小振幅极限环,本文主要研究六对称五次多项式微分系统:

$$\frac{dx}{dt} = (\frac{1}{2} - B_{50})y + (-\frac{1}{2} + 2B_{50})x^2y + (-1 - B_{50})x^4y + 2B_{41}x^3y^2 + (-\frac{1}{2} + 2B_{50})y^3 + (\frac{4}{3} - 2B_{50})x^2y^3 + (-\frac{1}{3} - B_{50})y^5 + \frac{1}{2}(2B_{41} - \frac{10W}{3})xy^4 + (B_{41} - \frac{3W}{2})x + (B_{41} - W)x^5 + (-2B_{41} + \frac{5W}{2})x^3 + (-2B_{41} + \frac{5W}{2})xy^2,$$

$$\frac{dy}{dt} = (-\frac{1}{2} + B_{50})x + (\frac{1}{2} - 2B_{50})x^3 + B_{50}x^5 + B_{41}x^4y + (\frac{1}{2} - 2B_{50})xy^2 + (2 + 2B_{50})\cdot x^3y^2 + (-\frac{2}{3} + B_{50})xy^4 + (2B_{41} - \frac{10W}{3})x^2y^3 + (B_{41} - \frac{3W}{2})y + (B_{41} - \frac{2W}{3})y^5 + (-2B_{41} + \frac{5W}{2})x^2y + (-2B_{41} + \frac{5W}{2})y^3 \quad (1)$$

的极限环分支问题,其中 $B_{41}, B_{50}, W \in R$,此系统总共有6个对称焦点 $(1,0), (\frac{1}{2}, \frac{\sqrt{3}}{2}), (-\frac{1}{2}, \frac{\sqrt{3}}{2}), (-1,0), (-\frac{1}{2}, -\frac{\sqrt{3}}{2}), (\frac{1}{2}, -\frac{\sqrt{3}}{2})$.通过奇点量和焦点量的计算,推出该系统在局部邻域内分支出12个极限环.

1 系统奇点量

由于系统(1)关于原点对称且有6个对称焦点 $(1,0), (\frac{1}{2}, \frac{\sqrt{3}}{2}), (-\frac{1}{2}, \frac{\sqrt{3}}{2}), (-1,0), (-\frac{1}{2},$

收稿日期: 2007-07-13
修回日期: 2007-07-27
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* 广西自然科学基金项目(0575092)资助。

$-\frac{\sqrt{3}}{2}$), ($\frac{1}{2}$, $-\frac{\sqrt{3}}{2}$). 不失一般性,仅讨论焦点(1,0)的情况即可. 首先把焦点(1,0)变换到原点. 通过变换 $x = u+1, y = v$,系统(1)变为系统:

$$\frac{du}{dt} = B_{41}(1+u)(2u+u^2+v^2)^2+\frac{1}{6}\{-3[2+10u+(13+8B_{50})u^2+8(1+B_{50})u^3+2(1+B_{50})u^4]v+[5-8(-2+3B_{50})u+(8-12B_{50})\cdot u^2]v^3-2(1+3B_{50})v^5-3u(-2+5u+15u^2+10u^3+2u^4)W+15(1+u)v^2W-10(1+u)v^4W\},$$

$$\frac{dv}{dt} = B_{50}u^5+u^4(5B_{50}+B_{41}v)+u^3[\frac{1}{2}+4B_{41}v+2v^2+2B_{50}(4+v^2)]+\frac{1}{6}u[6+3(13+8B_{50})\cdot v^2+(-4+6B_{50})v^4+8v^3(3B_{41}-5W)+30vW]+\frac{1}{6}v[15v+(-4+6B_{50})v^3+v^4(6B_{41}-4W)+6W-5v^2W]+u^2[\frac{3}{2}+4B_{41}v+6v^2+2B_{41}v^3+B_{50}(4+6v^2)+\frac{5vW}{2}-\frac{10v^3W}{3}]. \quad (2)$$

系统(2)$|_{W=0}$经过变换 $z = u+v\mathrm{i}, w = u-v\mathrm{i}$, $f = \mathrm{i}t, \mathrm{i} = \sqrt{-1}$,化为它的复伴随系统:

$$\frac{dz}{df} = z+a_{20}z^2+a_{02}w^2+a_{11}zw+a_{21}z^2w+a_{30}z^3+a_{12}zw^2+a_{03}w^3+a_{31}z^3w+a_{22}z^2w^2+a_{05}w^5+a_{32}z^3w^2,$$

$$\frac{dw}{df} = -(w+b_{20}w^2+b_{02}z^2+b_{11}wz+b_{21}w^2z+b_{30}w^3+b_{12}wz^2+b_{03}z^3+b_{31}w^3z+b_{22}w^2z^2+b_{05}z^5+b_{32}w^3z^2), \quad (3)$$

其中

$$a_{20} = 1-\mathrm{i}B_{41}+B_{50}, a_{02} = -3/2-\mathrm{i}B_{41}+B_{50}, a_{11} = 2(1-\mathrm{i}B_{41}+B_{50}), a_{30} = 1/6-\mathrm{i}B_{41}+B_{50}, a_{03} = -5/3, a_{21} = (3-8\mathrm{i}B_{41}+8B_{50})/2, a_{12} = (1-6\mathrm{i}B_{41}+6B_{50})/2, a_{31} = (1-6\mathrm{i}B_{41}+6B_{50})/3, a_{22} = (1-6\mathrm{i}B_{41}+6B_{50})/2, a_{05} = -1/6, a_{32} = 1/6-\mathrm{i}B_{41}+B_{50}, b_{TU} = \bar{a}_{TU}, T \geq 0, U \geq 0, T+U = 2,3,4,5. \quad (4)$$

根据文献[8]中定理2.2,得到计算系统(3)$|_{W=0}$在原点的递推公式.

定理1.1 对系统(3)$|_{W=0}$在原点的奇点量 $\_{}_m$, $m = 1,2\cdots$,可由递推公式 $c_{1,1} = 1, c_{0,2} = c_{2,0} = 0$确定. 当 $T = U$且 $U \neq 1$或 $T < 0$或 $U < 0$, $c(T,U) = 0$, 否则,

$$c(T,U) = \frac{1}{-T+U}(-b_{05}(1+U)c(-5+T,1+U)+(a_{50}(1+T)-b_{14}(1+U))c(-4+T,U)-b_{04}(1+U)c(-4+T,1+U)+(a_{41}(1+T)-b_{23}(1+U))c(-3+T,-1+U)+(a_{40}(1+T)-b_{13}(1+U))c(-3+T,U)-b_{03}(1+U)c(-3+T,1+U)+(a_{32}(1+T)-b_{32}(1+U))c(-2+T,-2+U)+(a_{31}(1+T)-b_{22}(1+U))c(-2+T,-1+U)+(a_{30}(1+T)-b_{12}(1+U))c(-2+T,U)-b_{02}(1+U)c(-2+T,1+U)+(a_{23}(1+T)-b_{41}(1+U))c(-1+T,-3+U)+(a_{22}(1+T)-b_{31}(1+U))c(-1+T,-2+U)+(a_{21}(1+T)-b_{21}(1+U))c(-1+T,-1+U)+(a_{20}(1+T)-b_{11}(1+U))c(-1+T,U)+(a_{14}(1+T)-b_{50}(1+U))c(T,-4+U)+(a_{13}(1+T)-b_{40}(1+U))c(T,-3+U)+(a_{12}(1+T)-b_{30}(1+U))c(T,-2+U)+(a_{11}(1+T)-b_{20}(1+U))c(T,-1+U)+a_{05}(1+T)c(1+T,-5+U)+a_{04}(1+T)c(1+T,-4+U)+a_{03}(1+T)c(1+T,-3+U)+a_{02}(1+T)c(1+T,-2+U)),$$

$$\_{}_m = -b_{05}c(-5+m,1+m)+(a_{50}-b_{14})c(-4+m,m)-b_{04}c(-4+m,1+m)+(a_{41}-b_{23})c(-3+m,-1+m)+(a_{40}-b_{13})c(-3+m,m)-b_{03}c(-3+m,1+m)+(a_{32}-b_{32})c(-2+m,-2+m)+(a_{31}-b_{22})c(-2+m,-1+m)+(a_{30}-b_{12})c(-2+m,m)-b_{02}c(-2+m,1+m)+(a_{23}-b_{41})c(-1+m,-3+m)+(a_{22}-b_{31})c(-1+m,-2+m)+(a_{21}-b_{21})c(-1+m,-1+m)+(a_{20}-b_{11})c(-1+m,m)+(a_{14}-b_{50})c(m,-4+m)+(a_{13}-b_{40})c(m,-3+m)+(a_{12}-b_{30})c(m,-2+m)+(a_{11}-b_{20})c(m,-1+m)+a_{05}c(1+m,-5+m)+a_{04}c(1+m,-4+m)+a_{03}c(1+m,-3+m)+a_{02}c(1+m,-2+m).$$

根据定理1.1的递推公式,利用计算机代数系统Mathematica来计算系统(3)在原点的奇点量.

定理1.2 系统(3)在原点的前2个奇点量为 $\_{}_1 = 8\mathrm{i}B_{41}B_{50}$, $\_{}_2 = 4\mathrm{i}B_{41}(1+12B_{41}^2)$,其中 $_{}_2$的表达式中已经令 $\_{}_1 = 0$.

由定理1.2知,系统(3)在原点的所有奇点量均为零当且仅当 $B_{41} = 0$. 实际上,如果 $B_{41} = 0$,那么有 $a_{kj} = b_{kj} \in R, kj \in \{02,20,11,30,03,21,12,31,22,05,32\}$,此时显然有,系统(3)所有基本Lie-不变量[7]对称且相等的. 根据广义对称原理[7]知,原点为系统(3)的中心,所以系统(3)在原点的奇点量均为零.

2 系统极限环分支

由于平移变换不改变系统的拓扑结构,故研究系统(1)的分支,只需要研究系统(2)的分支情况.

定理2.1 系统(3)原点为2阶细焦点,即 $\_{}_1 =$

$0, _{2} \neq 0$的充要条件是 $B_{50} = 0, B_{41} \neq 0$,且系统 (3) 原点的细焦点的最高阶数是 2阶.

定理 **2.2** 如果 W, B_{41}, B_{50} 满足 $W = -X_2, B_{50} = -X_1, B_{41} = 1$,其中 $X_i (i = 1, 2)$ 是满足 $0 < X_2 \ll X_1 \ll 1$,则系统 (3) 在原点充分小邻域内最多分支出 2 个极限环.

证明 由 $g_1(2c) = e^{2cW}, g_{2m+1}(2c) \sim i c\_{m}$,当定理 3.2条件满足时,

$$g_1(2c) - 1 \approx -2cX_2 + o(X_2),$$
$$g_3(2c) \approx [8c + k(X_1, X_2)]X_1 + o(X_1), \qquad (5)$$
$$g_5(2c) \approx -52c + o(1),$$

其中 $k(X_1, X_2)$ 在 (0,0) 解析且 $k(0,0) = 0$.由 (5)式可知 $g_{2(m-1)+1}(2c) g_{2m+1}(2c) < 0$且 $|g_{2(m-1)+1}(2c)| \ll |g_{2m+1}(2c)| (m = 1, 2)$.根据经典的 Bautin理论,系统 (2) 在原点充分小的邻域内最多可以分支出 2个极限环.根据系统 (1) 的对称性可以知道,在焦点 (1, 0), $(\frac{1}{2}, \frac{\sqrt{3}}{2})$, $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$, $(-1, 0)$, $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$, $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$ 处可以分别分支出 2个极限环.故系统在局部邻域内可以分支出 12个极限环.

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(责任编辑: 尹 闯)

强降雨量随着全球温度的增高而增多

科学家研究发现,随着全球暖化的持续,强降雨发生的频率增加了,而且它比目前的气候模型所预测的还要更加频繁。这些发现意味着,由于全球暖化而造成的对全球水循环变化的影响,比人们曾经想象的要严重许多。科学家们应用卫星观察资料和模型模拟来检验热带降雨和地球表面温度变化以及大气湿度之间的关系。无论是从卫星的数据还是从模型模拟的结果,他们的观察都披露了在气候变暖和极度降雨事件增加之间存在着直接的关联。然而,所观察到的极度降雨的增加比目前模拟所预测的增加要大得多,这表明对全球暖化造成的降雨量变化的预测可能被低估。科学家们认为,尽可能快地确定引起这种差异的原因,以及准确地了解全球暖化及其对水循环影响的意义,是至关重要的。

(据科学时报)