DOI: 10.13656/j.cnki.gxkx.2004.01.002
广西科学 Guangxi Sciences 2004, 11 (1): 4~ 9

M-泛函的经验似然置信区间*

Empirical Likelihood Confidence Intervals for *M*-Functionals

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摘要　利用经验似然思想，分别讨论不含附加信息和含附加信息时，*M*-泛函的经验似然置信区间，并推广 Zhang Biao (1997) 在独立同分布下的结果.

关键词　*M*-泛函　经验似然　置信区间　附加信息
中图法分类号　O212.2

Abstract　We employ the method of empirical likelihood to construct the confidence intervals for *M*-functionals in the presence of auxiliary information and without auxiliary information, and extend Zhang Biao's (1997) results.

Key words　*M*-functionals, empirical likelihood, confidence intervals, auxiliary information

1 定义

M-泛函　$X_1,\cdots,X_n$ 为强平稳 O-混合 (见文献 [1] 中定义) 的随机变量, 分布函数为 F, M-泛函 $\theta(F)$ 定义为下面方程的根

$$E_F h(X,\theta_0) = \int h(x,\theta_0)\mathrm{d}F(x) = 0, X\sim F, x,\theta_0, h(x,\theta_0)\in R. \quad (1.1)$$

(a) 不带附加信息时的经验似然比统计量

$$E_F h(X,\theta_0) = 0.$$

在约束条件下,

$$\sum_{i=1}^{n} p_i = 1, p_i \geqslant 0, \sum_{i=1}^{n} p_i h(X_i,\theta_0) = 0,$$

则经验似然比统计量为

$$l_1(\theta_0) = 2\sum_{i=1}^{n}\log[1+\lambda h(X_i,\theta_0)], \quad (1.2)$$

其中 λ 满足

$$\frac{1}{n}\sum_{i=1}^{n}\frac{h(X_i,\theta_0)}{1+\lambda h(X_i,\theta_0)} = 0. \quad (1.3)$$

(b) 含附加信息时的经验似然比统计量

$$E_F g(X) = 0, g(x) = (g_1(x),\cdots,g_r(x))', \quad (1.4)$$

令 $h(x,\theta_0) = (g'(x), O(x,\theta_0))'$, 易知 $E_F h(X,\theta_0) = 0$, 由 Zhang Biao(1997) 文献中, 可知经验似然比统计量为[2]

$$l_2(\theta_0) = 2\sum_{i=1}^{n}\log[1+\lambda' h(X_i,\theta_0)] - 2\sum_{i=1}^{n}\log[1+Z g(X_i)], \quad (1.5)$$

其中 $\lambda = (\lambda_1,\cdots,\lambda_{r+1})'$ 满足

$$\frac{1}{n}\sum_{i=1}^{n}\frac{h(X_i,\theta_0)}{1+\lambda' h(X_i,\theta_0)} = 0. \quad (1.6)$$

$Z = (Z_1,\cdots,Z_r)'$ 满足

$$\frac{1}{n}\sum_{i=1}^{n}\frac{g(X_i)}{1+Z g(X_i)} = 0. \quad (1.7)$$

经验似然是由 Owen 引入的一种非参数推断方法[3~5], Zhang Biao 已经讨论了在有附加信息时 M-泛函的经验似然置信区间[2], 并假设 $X_1, X_2,\cdots,X_n$ 为独立同分布的随机变量, 本文假设 $X_1,\cdots,X_n$ 为强平稳 O-混合随机变量, 讨论有附加信息及不带附加信息时, M-泛函的经验似然置信区间. 本文仅给出似然比统计量的极限分布.

2 引理

引理 1　设 $\{X_i\}$ 为强平稳 O-混合随机变量序列, 满足 $\sum_{k=1}^{\infty} O^{\frac{1}{2}}(k) < \infty$, 假定 $E|X_1|^2 < \infty$, $EX_1 = 0$, 则 $A^2 = EX_1^2 + 2\sum_{i=1}^{\infty}E(X_1X_{1+i})$ 收敛且有

$$\sup_{-\infty<x<\infty}\left|P\left\{\frac{n^{-\frac{1}{2}}\sum_{j=1}^{n}X_j}{A} < x\right\} - H(x)\right| \to 0,$$

2003-09-17收稿, 2003-10-28修回.
* Partly supported by the NSF of China (10161004) and the SF of Guangxi Normal University (0007014).

a. s.

注 该引理为文献[6]中系 4.7的特例.

引理 2 $Y_1,\cdots,Y_n$ 为同分布的随机变量, $Z_n=\max_{1\le i\le n}|Y_i|$, 若 $E|Y_1|^s<\infty$, 则 $Z_n=o(n^{1/s})$, a. s.

注 参见文献[7].

引理 3 令 $U_1,\cdots,U_n$ 是一元 ρ-混合随机变量, $E|U_i|^r<\infty$, 对于 $i=1,\cdots,n$ 及某个 $r\ge 2$, 则

$$E|\sum_{i=1}^{n}U_i|^r\le Cn^{r/2}\max_{1\le i\le n}E|U_i|^r.$$

证明 参见文献[8]中定理 2.3的证明.

引理 4 令 $\{X_{nj}\}$ 是强平稳 ρ-混合三角阵列, $\rho(1)<1, \sum_{j=1}^{\infty}\rho^{1/2}(j)<\infty$, 假设 $\{X_{nj}\}\in R$, 并且

(1) $EX_{n1}=0, EX_{n1}^2<\infty$,

(2) 对于任意的 $\varepsilon>0$, $\lim_n nE[X_{n1}^2I(|X_{n1}|>\varepsilon)]=0$,

(3) $\sup_n nEX_{n1}^2<\infty$, $A=\lim_n\{nEX_{n1}^2+2n\sum_{j=1}^{n-1}E(X_{n1}X_{n,j+1})\}$ 存在, $A>0$, 则有 $\sum_{j=1}^{n}X_{nj}\to_d N(0,A)$.

注 强平稳 ρ-混合三角阵列定义参考 Samur[6], 此引理为文献[6]中命题 4.6的特例.

引理 5[9] 设 $\{U_i,i\ge 1\}$ 是一元 ρ-混合随机变量, $E|U_i|^p<\infty$, $i,j\ge 1$, $p,q>1$, $1/p+1/q=1$, 则

$$|EU_iU_{i+j}-EU_iU_{i+j}|\le 2\rho^{1/p}(j)E^{1/p}|U_i|^pE^{1/q}|U_{i+j}|^q.$$

引理 6 如果 $\Sigma_g=E[g(X)g'(X)]$ 是正定的, $\sum_{k=1}^{\infty}\rho^{1/2}(k)<\infty$, $E\|g(X)\|^4<\infty$, 则 $\lambda=G^{-1}\bar g+o_p(n^{-1/2})$, 其中 $G=\frac{1}{n}\sum_{i=1}^{n}g(X_i)g'(X_i)$, $\bar g=\frac{1}{n}\sum_{i=1}^{n}g(X_i)$.

证明 令 $\lambda=d\theta$, 其中 $d=\|\lambda\|$, $\|\theta\|=1$,

$$G=\frac{1}{n}\sum_{i=1}^{n}g(X_i)g'(X_i),\ \bar g=\frac{1}{n}\sum_{i=1}^{n}g(X_i),$$

$$0=\|g_1(d\theta)\|\ge|\theta'g_1(d\theta)|=\frac{1}{n}|\theta'\sum g(X_i)-d\sum\frac{g(X_i)\theta'g(X_i)}{1+d\theta'g(X_i)}|\ge\frac{d}{n}\theta'\sum\frac{g(X_i)g'(X_i)}{1+d\theta'g(X_i)}\theta-\frac{1}{n}|\sum_{j=1}^{r}e_j\sum g(X_i)|\ge\frac{d\theta'G\theta}{1+dZ_n}-\frac{1}{n}|\sum_{j=1}^{r}e_j\sum g(X_i)|,$$

其中 $Z_n=\max_{1\le i\le n}\|g(X_i)\|$, 其中 e_j 的第 j 个分量与 θ 的相应分量相同, 其余分量为 0.

由引理 1, $\forall a\in R^r, a\neq 0$,

$$\sup_{-\infty<x<\infty}|P\{\frac{n^{-1/2}\sum a'g(X_i)}{A}<x\}-H(x)|\to 0,\ \text{a. s.}$$

故 $\frac{1}{n}\sum a'g(x_i)=O_p(1)$,

$$a'\bar g=O_p(n^{-1/2}),\qquad(2.1)$$

式中, $A^2=Ea'g(X_1)g'(X_1)a+2\sum_{i=1}^{\infty}E(a'g(X_1)g'(X_{1+i})a)=a'[Eg(X_1)g'(X_1)+2\sum_{i=1}^{\infty}E(g(X_1)g'(X_{1+i}))]a$,

记 $A_1=Eg(x_1)g'(x_1)+2\sum_{i=1}^{\infty}E(g(X_1)g'(X_{1+i}))$,

即 $\frac{\sqrt{n}a'\bar g}{\sqrt{a'A_1a}}\to_d N(0,1)$,

$$\sqrt{n}\bar g\to_d N(0,A_1).\qquad(2.2)$$

下证 $G=\Sigma_g+O_p(1)$.

$$G-\Sigma_g=\frac{1}{n}\sum_{i=1}^{n}[g(X_i)g'(X_i)-Eg(X_i)g'(X_i)],$$

$$E(G_{kl}-\Sigma_{gkl})^2=\frac{1}{n^2}E[\sum_{i=1}^{n}(g_k(X_i)g_l(X_i)-Eg_k(X_i)g_l(X_i))]^2\le\frac{C}{n^2}nE(g_k(X_1)g_l(X_1)-Eg_k(X_1)g_l(X_1))^2\le\frac{C}{n}E(g_k(X_1)g_l(X_1))^2\le\frac{C}{n}[Eg_k^4(X_1)+Eg_l^4(X_1)]\to 0.$$

故 $$G=\Sigma_g+o_p(1).\qquad(2.3)$$

故 $\theta'G\theta=\theta\Sigma_g\theta+o_p(1)\ge\sigma_p+o_p(1)$.

由引理 2, $Z_n=o_p(n^{1/2})$.

$$\frac{d}{1+dZ_n}=O_p(n^{-1/2})$$

$$d=\frac{O_p(n^{-1/2})}{1-Z_nO_p(n^{-1/2})}=\frac{O_p(n^{-1/2})}{1+o_p(1)}=O_p(n^{-1/2}).$$

即 $d=\|\lambda\|=O_p(n^{-1/2})$.

令 $\gamma_i=\lambda'g(X_i)$, 则

$$\max_{1\le i\le n}|\gamma_i|=\max_{1\le i\le n}|\lambda'g(X_i)|\le\|\lambda\|\max_{1\le i\le n}\|g(X_i)\|=O_p(n^{-1/2})o_p(n^{1/2})=o_p(1).$$

$$g_1(\lambda)=\frac{1}{n}\sum_{i=1}^{n}\frac{g(X_i)}{1+\lambda'g(X_i)}=0,$$

$$\lambda=G^{-1}\bar g+G^{-1}\frac{1}{n}\sum_{i=1}^{n}g(X_i)\gamma_i^2/(1+\gamma_i).$$

$$\frac{1}{n}\|\sum_{i=1}^{n}g(X_i)\gamma_i^2/(1+\gamma_i)\|\le\frac{1}{n}\sum_{i=1}^{n}\|g(X_i)\gamma_i^2/(1+\gamma_i)\|\le\frac{1}{n}\sum_{i=1}^{n}\|g(X_i)\|^3\|\lambda\|^2(1+\gamma_i)^{-1}=o_p(n^{1/2})O_p(n^{-1})O_p(1)=o_p(n^{-1/2}).$$

故 $\lambda=G^{-1}\bar g+o_p(n^{-1/2})$.

引理 7 假设 $\theta_0=\theta(F)$ 满足(1.1)式, 存在且唯一, $\psi(x,\theta)$ 关于 x 可测, $\Sigma_h=E[h(X,\theta_0)h'(X,\theta_0)]$

是正定,$\sum_{k=1}^{\infty} O^{1/2}(k) < \infty$,$O(1) < 1, E\| h(X,\theta_0)\|^4 < \infty$,$\frac{\partial h(x,\theta)}{\partial\theta}$ 在 θ_0 点连续,$|\frac{\partial j(x,\theta)}{\partial\theta}| \leqslant M(x)$,$|\theta - \theta_0| < X$,$EM^4(X) < \infty$,则对于 $\theta = \theta_0 + O_p(n^{-1/2})$,$\lambda = H_\theta^{-1}\bar{h}_\theta + o_p(n^{-1/2})$,其中 $H_\theta = \frac{1}{n}\sum_{i=1}^{n} h(X_i,\theta)h'(X_i,\theta)$,$\bar{h}_\theta = \frac{1}{n}\sum_{i=1}^{n} h(X_i,\theta)$.

证明 令 $\lambda = dU$,其中 $d = \|\lambda\|$,$\|U\| = 1$.

$\theta = \| g_2(dU)\| \geqslant |U' g_2(dU)| = \frac{1}{n}|U'\sum h(X_i,\theta) - d\sum \frac{h(X_i,\theta)U'h(X_i,\theta)}{dU'h(X_i,\theta)}| \geqslant \frac{d}{n}U'\cdot$

$\sum \frac{h(X_i,\theta)h'(X_i,\theta)}{1+dU'h(X_i,\theta)}U - \frac{1}{n}|\sum_{j=1}^{r+1} e_j \sum h(X_i,\theta)| \geqslant \frac{dU'H_\theta U}{1+dZ_n'} - \frac{1}{n}|\sum_{j=1}^{r+1} e_j\sum h(X_i,\theta)|$,

其中 e_j 的第 j 个分量与 U的相应分量相同,其余为 0,$Z_n' = \max_{1\leqslant i\leqslant n}\| h(X_i,\theta)\|$. $\forall a \neq 0, a \in R^{r+1}$,令 $X_{ni} = \frac{1}{\sqrt{n}}a'(h(X_i,\theta) - Eh(X_i,\theta))$.

(1) $EX_{n1} = 0$,

$EX_{n1}^2 = \frac{1}{n}E[a'(h(X_1,\theta) - Eh(X_1,\theta))]^2 \leqslant \frac{1}{n}\| a\|^2 E\| h(X_1,\theta)\|^2 \leqslant \frac{1}{n}\| a\|^2 [E(g_1^2(X_1) + \cdots + g_r^2(X_1) + h^2(X_1,\theta)) + CEM^2(X_1) + CEM(X_1)] < \infty$.

(2) $\forall X > 0$,

$nE[X_{n1}^2 I[|X_{n1}| > X]] \leqslant n\frac{1}{X}EX_{n1}^4 = \frac{1}{nX^2}E[a'(h(X_1,\theta) - Eh(X_1,\theta))]^4$.

$I = \frac{1}{n}E[a'(h(X_1,\theta) - Eh(X_1,\theta))]^4 \leqslant \frac{1}{n}\| a\|^4 E\| h(X_1,\theta)\|^4 \leqslant \frac{C}{n}(\sum_{i=1}^{r} Eg_i^4(X_1) + Eh^4(X_1,\theta)) \to 0$.

(3) $nEX_{n1}^2 = E[a'(h(X_1,\theta) - Eh(X_1,\theta))]^2 = a'E(h(X_1,\theta) - Eh(X_1,\theta))(h(X_1,\theta) - Eh(X_1,\theta))'a \to a'Eh(X_1,\theta_0)h'(X_1,\theta_0)a$.

$n\sum_{j=1}^{n-1}E(X_{n1}X_{n,j+1}) = \sum_{j=1}^{n-1}a'E(h(X_1,\theta) - Eh(X_1,\theta))(h(X_{1+j},\theta) - Eh(X_{1+j},\theta))'a$.

令 $a_j = a'Eh(X_1,\theta_0)h'(X_{1+j},\theta_0)a$,$a_{nj} = a'E(h(X_1,\theta) - Eh(X_1,\theta))(h(X_{1+j},\theta) - Eh(X_{1+j},\theta))'a$.

先证 $\sum_{j=1}^{n-1}|a_j|$ 收敛.

$\sum_{j=1}^{n-1}|a_j| \leqslant C\sum_{j=1}^{n-1}O^{1/2}(j)$,

故 $\sum_{j=1}^{\infty}a_j$ 收敛,即 $\sum_{j=1}^{\infty}a'Eh(X_1,\theta_0)h'(X_{1+j},\theta_0)a$ 收敛.

由引理 5,

$\sum_{j=1}^{n-1}|a_{nj}| \leqslant C\sum_{j=1}^{n-1}O^{1/2}(j)$,

$\forall X > 0, \exists N, N_1$,使得当 $n > \max(N, N_1)$ 时,$\sum_{N+1}^{\infty}O^{1/2}(j) < X$,$|a_{nj} - a_j| \leqslant X$.

$|\sum_{i=N+1}^{n}a_{ni}| \leqslant \sum_{i=N+1}^{n}|a_{ni}| \leqslant X$,

$|\sum_{i=N+1}^{n}a_i| \leqslant X$,

$|\sum_{i=1}^{n}a_{ni} - \sum_{i=1}^{n}a_i| \leqslant |\sum_{i=1}^{N}a_{ni} - \sum_{i=1}^{N}a_i| + |\sum_{i=N+1}^{n}a_{ni} - \sum_{i=N+1}^{n}a_i| \leqslant \sum_{i=1}^{n}|a_{ni} - a_i| + 2X \leqslant (N+2)X$.

$$\lim_{n\to\infty}\sum_{i=1}^{n}a_{ni} = \sum_{i=1}^{\infty}a_i. \tag{2.4}$$

故 $A = a'[Eh(X_1,\theta_0)h'(X_1,\theta_0) + 2\sum_{j=1}^{\infty}Eh(X_1,\theta_0)h'(X_{1+j},\theta_0)]a$,且 $A > 0$.

由引理 4,$\sum_{j=1}^{n}X_{nj} \to_d N(0,A)$.

$\frac{1}{\sqrt{n}}\sum_{j=1}^{n}a'(h(X_j,\theta) - Eh(X_j,\theta)) \to_d N(0,A)$.

$\sqrt{n}\,a'\bar{h}_\theta - \sqrt{n}\,a'Eh(X_1,\theta) = O_p(1)$.

$a'\bar{h}_\theta = O_p(n^{-1/2})$.

$\sqrt{n}(\bar{h}_\theta - E\bar{h}_\theta) \to_d N(0, A_2)$.

$A_2 = Eh(X_1,\theta_0)h'(X_1,\theta_0) + 2\sum_{j=1}^{n}Eh(X_1,\theta_0)h'(X_{1+j},\theta_0)$.

$$\sqrt{n}\,\bar{h}_\theta \to_d N(b, A_2), b = (0, e_h W)'. \tag{2.5}$$

下证 $H_\theta = Eh_\theta + o_p(1)$.

$H_\theta - EH_\theta = \frac{1}{n}\sum_{i=1}^{n}[h(X_i,\theta)h'(X_i,\theta) - Eh(X_i,\theta)h'(X_i,\theta)]$.

$E(H_{\theta_{kl}} - EH_{\theta_{kl}})^2 = \frac{1}{n^2}E[\sum_{i=1}^{n}(h_k(X_i,\theta)h_l(X_i,\theta) - Eh_k(X_i,\theta)h_l(X_i,\theta))]^2 \leqslant \frac{C}{n^2}nE(h_k(X_1,\theta)h_l(X_1,\theta))^2 = \frac{C}{n}E(h_k(X_1,\theta)h_l(X_1,\theta))^2 \leqslant \frac{C}{n}[Eh_k^4(X_1,\theta) + Eh_l^4(X_1,\theta)] \to 0$.

故 $H_\theta = EH_\theta + o_p(1)$.

又 $H_\theta = \sum h + o(1)$.

故 $H_\theta = \sum h + o_p(1)$.

$U'H_\theta U \geqslant e_h + o_p(1)$,$Z_n' = \max_{1\leqslant i\leqslant n}\| h(X_i,\theta)\| = o_p(n^{1/4})$.

$\frac{d}{1+dZ_n'} = O_p(n^{-1/2})$,$d = \| Z\| = O_p(n^{-1/2})$.

同理,令 $V_i = Z'h(X_i,\theta)$,则

$\max_{1\leqslant i\leqslant n}|V_i| = o_p(1)$,

$$Z = H_0^{-1}\bar{h}_0 + H_0^{-1}\frac{1}{n}\sum_{i=1}^{n} h(X_i,\theta)V_i^2/(1+V_i).$$

引理 8 如果 A 是 $(p+q)\times(p+q)$ 满秩方阵，$A=\begin{pmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}$，$A_{11}$ 是 $p\times p$ 满秩方阵，A_{22} 是 $q\times q$ 满秩方阵，则 $(A^{-1}-B_1)A$ 为秩 q 的幂等阵，其中 $B_1=\begin{pmatrix}A_{11}^{-1} & O\\ O & O\end{pmatrix}$.

证明 $(A^{-1}-B_1)A = I - B_1A$.

$$B_1A=\begin{pmatrix}I_{p\times p} & A_{11}^{-1}A_{12}\\ O & O\end{pmatrix},$$

$$(B_1A)^2=\begin{pmatrix}I_{p\times p} & A_{11}^{-1}A_{12}\\ O & O\end{pmatrix}\begin{pmatrix}I_{p\times p} & A_{11}^{-1}A_{12}\\ O & O\end{pmatrix}=\begin{pmatrix}I_{p\times p} & A_{11}^{-1}A_{12}\\ O & O\end{pmatrix}=B_1A,$$

故 B_1A 为幂等阵.

$$[(A^{-1}-B_1)A]^2=(I-B_1A)(I-B_1A)=I-2B_1A+(B_1A)^2=I-B_1A=(A^{-1}-B_1)A.$$

则 $(A^{-1}-B_1)A$ 为幂等阵.

$$I-B_1A=\begin{pmatrix}O_{p\times p} & A_{11}^{-1}A_{12}\\ O & I_{q\times q}\end{pmatrix},$$

秩为 q.

3 定理

定理 1 假设 $\theta_0=\theta(F)$ 满足 (1.1) 式，存在且唯一，$h(x,\theta)$ 关于 x 可测，进一步假设 $\lambda_F(\theta_0)$ 有限且非 0，$h(x,\theta)$ 在 θ_0 点关于 x 一致连续，$\frac{\partial h(x,\theta)}{\partial\theta}$ 在 θ_0 连续，$\left|\frac{\partial h(x,\theta)}{\partial\theta}\right|\leqslant M(x)$，在 θ_0 邻域内，$EM(X)^4<\infty$，对于正数 d，常数 W，有

$$\lim_{n\to\infty}P(l_1(\theta)\leqslant d)=P\left(\frac{A}{e_h^2}\chi^2_{(1)}(e_h^2W^2)\leqslant d\right),$$

由于 A 未知，所以此结论并不理想，需要对它改进去掉其中的参数，为此对其进行分组，令

$$p'=[n^{T}],\ 0<T\leqslant 1/3,\ q'=\left[\frac{n}{p'}\right],$$

$$k_i=\frac{1}{p'}\sum_{j=1}^{p'}h(X_{(i-1)p'+j}),\ i=1,\cdots,q',$$

则分组后经验似然比统计量

$$\lim_{n\to\infty}P(\hat{l}_1(\theta)\leqslant d)=P(\chi^2_{(1)}(e_hW_2)\leqslant d),$$

其中 $\theta=\theta_0+\frac{e_h}{\sqrt{n}\lambda_F(\theta_0)}W$，$e_h^2=Eh^2(X,\theta_0)$，$\lambda_F(\theta_0)=E[\partial h(X,\theta)/\partial\theta|_{\theta=\theta_0}]$.

定理 2 假设 $\theta_0=\theta(F)$ 满足 (1.1) 式，存在且唯一，$h(x,\theta)$ 关于 x 可测，进一步假设 $\lambda_F(\theta_0)$ 有限且非 0，$h(x,\theta)$ 在 θ_0 点关于 x 一致连续，$\frac{\partial h(x,\theta)}{\partial\theta}$ 在 θ_0 连续，$\left|\frac{\partial h(x,\theta)}{\partial\theta}\right|\leqslant M(x)$，在 θ_0 邻域内，$EM(X)^4<\infty$，对于正数 d，常数 W，有

$$\lim_{n\to\infty}P(l_2(\theta)\leqslant d)=P(Z'BZ\leqslant d),$$

其中 $\theta=\theta_0+\frac{e_h}{\sqrt{n}\lambda_F(\theta_0)}W$，$Z\sim N(a,A_2)$，$a=(0',e_hW)'$，$B=\sum_h^{-1}-\begin{pmatrix}\sum_g^{-1} & O\\ O & 0\end{pmatrix}$.

由于我们不知道 A_2，所以使用分组经验似然来克服一般经验似然之不足，令

$$p=[n^{T}],\ 0<T\leqslant 1/3,\ q=\left[\frac{n}{p}\right],$$

$$k_{1i}=\frac{1}{p}\sum_{j=1}^{p}g(X_{(i-1)p+j}),\ i=1,\cdots,q,$$

$$k_{2i}=\frac{1}{p}\sum_{j=1}^{p}h(X_{(i-1)p+j},\theta_0),\ i=1,\cdots,q,$$

则似然比统计量为

$$\hat{l}_2(\theta_0)=2\sum_{i=1}^{q}\log(1+\lambda'k_{2i})-2\sum_{i=1}^{q}\log(1+Z'k_{1i}). \quad (3.1)$$

其中 λ 满足

$$\frac{p}{q}\sum_{j=1}^{q}\frac{k_{2i}}{1+\lambda'k_{2i}}=0, \quad (3.2)$$

Z 满足

$$\frac{p}{q}\sum_{j=1}^{q}\frac{k_{1i}}{1+Z'k_{1i}}=0. \quad (3.3)$$

定理 3 假设同定理 2，

$$\lim_{n\to\infty}P(\hat{l}_2(\theta)\leqslant d)=P(\chi_1^2(d^2)\leqslant d),$$

其中

$$d^2=\frac{e_h^2W^2}{e_h^2+2\sum_{j=1}^{\infty}Eh(X_1,\theta_0)h(X_{1+j},\theta_0)-B_2A_1^{-1}B_2'},$$

$$B_2=Eh(X,\theta_0),g'(X_1)+2\sum_{j=1}^{\infty}Eh(X_{1+j},\theta_0),g'(X_1).$$

4 定理证明

定理 1 证明 令 $h=\frac{1}{n}\sum_{i=1}^{n}h^2(X_i,\theta)$，$\bar{h}(\theta)=\frac{1}{n}\sum_{i=1}^{n}h(X_i,\theta)$，$k_i=\max_{1\leqslant i\leqslant n}|h(X_i,\theta)|$，$|h(X_i,\theta)|=|h(X_i,\theta_0)+O(n^{-1/2})\frac{\partial h((X_i,Y)}{\partial\theta}|$，

故

$$\max_{1\leqslant i\leqslant n}|h(X_i,\theta)|\leqslant|h(X_i,\theta_0)|+O(n^{-1/2})\max_{1\leqslant i\leqslant n}|M(X_i)|,$$

因 $Eh^2(X,\theta_0)<\infty$，故

$$\max_{1\leqslant i\leqslant n}|h(X_i,\theta_0)|=o_p(n^{1/2}).$$

又 $EM^4(X_i)<\infty$，

$O(n^{-1/2})\max_{1\leqslant i\leqslant n}|M(X_i)| = o_p(n^{-1/4})$,

故 $\max_{1\leqslant i\leqslant n}|h(X_i,\theta)| = o_p(n^{1/2})$. (4.1)

下证 $\bar h_1 = E\bar h_1 + o_p(1)$.

$$\bar h_1 - E\bar h_1 = \frac{1}{n}\sum_{i=1}^{n}[h^2(X_i,\theta) - Eh^2(X_i,\theta)],$$

$$E(\bar h_1 - E\bar h_1)^2 = \frac{1}{n^2}E[\sum_{i=1}^{n}(h^2(X_i,\theta) - Eh^2(X_i,\theta))]^2,$$

$$E(\bar h_1 - E\bar h_1)^2 \leqslant \frac{C_n}{n^2}E(h^2(X_1,\theta) - Eh^2(X_1,\theta))^2 \leqslant \frac{C}{n}Eh^4(X_1,\theta) \to 0,$$

故 $\bar h_1 = E\bar h_1 + o_p(1)$.

而 $E\bar h_1 = Eh^2(X_1,\theta_0) + o(1)$,故

$$\bar h_1 = Eh^2(X_1,\theta_0) + o_p(1). \quad (4.2)$$

令 $X_{ni} = \frac{1}{\sqrt{n}}(h(X_i,\theta) - Eh(X_i,\theta))$,

(1) $EX_{n1} = 0$, $EX_{n1}^2 = \frac{1}{n}E(h(X_1,\theta) - Eh(X_1,\theta))^2 \leqslant \frac{1}{n}Eh^2(X_1,\theta) < \infty$.

(2) $\forall \varepsilon > 0$,

$$nE[X_{n1}^2 I[|X_{n1}| > \varepsilon]] \leqslant \frac{n}{\varepsilon^2}EX_{n1}^4 = \frac{1}{\varepsilon^2}\frac{1}{n}E(h(X_i,\theta) - Eh(X_1,\theta))^4,$$

记 $I = \frac{1}{n}E(h(X_1,\theta) - Eh(X_1,\theta))^4 \leqslant \frac{1}{n}Eh^4(X_1,\theta) + \frac{1}{n}[Eh(X_1,\theta)]^4 \to 0$.

(3) $nEX_{n1}^2 = E[h(X_1,\theta) - Eh(X_i,\theta)]^2 \to Eh^2(X_1,\theta_0)$.

$$n\sum_{j=1}^{n-1}E(X_{n1}X_{n,j+1}) = \sum_{j=1}^{n-1}[Eh(X_1,\theta)h(X_{1+j},\theta) - Eh(X_1,\theta)Eh(X_{1+j},\theta)] \to \sum_{j=1}^{\infty}Eh(X_1,\theta_0)h(X_{1+j},\theta_0),$$ 故

$$A = Eh^2(X_1,\theta_0) + 2\sum_{j=1}^{\infty}Eh(X_1,\theta)h(X_{1+j},\theta_0),$$

$A > 0$.

由引理 4, $\sum_{j=1}^{n}X_{nj} \to_d N(0,A)$.

即 $\frac{1}{\sqrt{n}}\sum_{j=1}^{n}[h(X_j,\theta) - Eh(X_j,\theta)] \to_d N(0,A)$.

$$\sqrt{n}\,\bar h - \sqrt{n}\,Eh(X,\theta) \to_d N(0,A).$$

故 $\sqrt{n}\,\bar h = O_p(1)$,

$\bar h = O_p(n^{-1/2})$,

$$\sqrt{n}\,\bar h \to_d N(b,A), \quad (4.3)$$

其中 $b = \sigma_h\omega$,故

$$l_1(\theta) = ng^{-1}\bar h^2 + o_p(1) \to \frac{A}{\sigma_h^2}\chi^2_{(1)}(\sigma_h^2\omega^2).$$

证毕.

定理 2证明

$$l_2(\theta) = 2\sum_{i=1}^{n}\log[1 + \lambda' h(X_i,\theta)] - 2\sum_{i=1}^{n}\log[1 + Z' g(X_i)] = n\bar h'_{\theta} H_{\theta}^{-1}\bar h_{\theta} - n\bar g' G^{-1}\bar g + o_p(1) = n\bar h'_{\theta}\sum\nolimits_{\theta}^{-1}\bar h_{\theta} - n\bar g\sum\nolimits_{g}^{-1}\bar g + o_p(1) = n\bar h'_{\theta}B\bar h_{\theta} + o_p(1) = (\sqrt{n}\,\bar b_0)'B(\sqrt{n}\,\bar b_0) + o_p(1) \to Z'BZ.$$

其中 $B = \sum\nolimits_h^{-1} - \begin{pmatrix}\sum\nolimits_g^{-1} & O\\ O & 0\end{pmatrix}$, $\bar h(\theta) = \frac{1}{n}\sum_{i=1}^{n}h(X_i,\theta)$, $\bar b_0 = (\bar g', \bar h(\theta_0) + \lambda_F(\theta_0)(\theta - \theta_0))'$, $\bar h_0 = \bar b_0 + o_p(n^{-1/2})$, $Z \sim Z(a, A_2)$, $a = (0', \sigma_h\omega)'$.

证毕.

定理 3证明 仿定理 2证明,知

$$\sqrt{pq}\,\bar\xi_1 \to_d N(0, A_1), \quad (4.4)$$

$$a'\bar\xi_1 = O_p((pq)^{-1/2}), \quad (4.5)$$

$$Z_q = \max_{1\leqslant j\leqslant q}\|\xi_{1j}\| = o_p((q/p)^{1/2}). \quad (4.6)$$

其中 $G_{(1)} = \frac{p}{q}\sum\xi_{1j}\xi'_{1j}$, $\bar\xi_1 = \bar g$, $G_{(1)} - EG_{(1)} = \frac{p}{q}\sum(\xi_{1j}\xi'_{1j} - E\xi_{1j}\xi'_{1j})$,

$$E[(G_{(1)} - EG_{(1)})_{kl}]^2 \leqslant C\frac{p^2}{q^2}q\max_{1\leqslant i\leqslant q}E(\xi_{1i,k}\xi_{1i,l} - E\xi_{1i,k}\xi_{1i,l})^2 \leqslant C\frac{p^2}{q^2}E(\xi_{11,k}\xi_{11,l})^2 \leqslant C\frac{p^2}{q^2}(E\xi_{11,k}^4 + E\xi_{11,l}^4) \leqslant \frac{C}{q}Eg_k^4(X_1) \leqslant \frac{C}{q} \to 0.$$

故 $G_{(1)} = EG_{(1)} + \sqrt{pq}\,\bar\xi_2$.

而 $EG_{(1)} = \frac{p}{q}\sum_{i=1}^{q}E\xi_{1i}\xi'_{1i} = pE\xi_{11}\xi'_{11} = \frac{1}{p}E(\sum_{j=1}^{p}g(X_j)\sum_{j=1}^{p}g'(X_j))$,

$EG_{(1)} = A_1 + o(1)$.

同理可证 $EH_{\theta 1} = A_2 + o(1)$. (4.7)

$$\hat{\mathscr{L}}_2(\theta) = pq\bar\xi'_2 A_2^{-1}\bar\xi_2 - pq\bar\xi'_1 A_1^{-1}\bar\xi_1 + o_p(1) = pq\bar\xi_2 B'\bar\xi_2 + o_p(1) = (\sqrt{pq}\,\bar b_0)'B'(\sqrt{pq}\,\bar b_0) + o_p(1) \to Z'B'Z = \chi_1^2(\delta^2),$$

其中 $B' = A_2^{-1} - \begin{pmatrix}A_1^{-1} & O\\ O & 0\end{pmatrix}$, $Z \sim N(a, A_2)$,

$$\delta^2 = a'B'a = \frac{\sigma_h^2\omega^2}{\sigma_h^2 + 2\sum_{j=1}^{\infty}Eh(X_1,\theta_0)h(X_{1+j},\theta_0) - B_2A_1^{-1}B'_2} > \frac{\sigma_h^2\omega^2}{\sigma_h^2 + 2\sum_{j=1}^{\infty}Eh(X_1,\theta_0)h(X_{1+j},\theta_0)},$$

其中

$$B_2 = Eh(X_1,\theta_0)g'(X_1) + 2\sum_{j=1}^{\infty} Eh(X_{1+j},\theta_0)g'(X_1),$$

$$B'_2 = Eg(X_1)h(X_1,\theta_0) + 2\sum_{j=1}^{\infty} Eg(X_1)h(X_{1+j},\theta_0).$$

证毕.

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