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大系统优化的动态规划递阶算法改进

Improvement of Hierachical Algorithm for Large Scale System Dynamic Programming

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摘要　引入 1个拉格朗日乘子,将动态规划的最优性原则与大系统控制论中的分解协调的递阶算法结合起来,并考虑各子系统间的顺序直接作用,提出 1种以二次型为性能指标的离散线性系统优化算法,并导出各子系统的递推公式.

关键词　大系统　动态规划　拉格朗日乘子　算法
中图法分类号　O231.1

Abstract　An algorithm for optimizing discrete linear system with a quadratic cost function is proposed by importting one Lagrange multiplier. The approaching formulae for sub-systems are developed.

Key words　large scale system, dynamic programming, Lagrange multiplier, algorithm

文献 [1]中引入 2个拉格朗日乘子,将动态规划的最优性原则与大系统控制论中的分解协调的递阶算法结合起来,考虑到各子系统间的顺序直接作用,提出了一种算法,并导出了各子系统的递推公式,本文引入 1个拉格朗日乘子,与文献 [1]相比,计算量相对减少,只有 1个协调变量,减少了搜索次数.

1　系统的分解

设离散线性系统的状态方程如下:

$$X(k+1) = A(k)X(k) + B(k)U(k), \tag{1}$$

$$X(0) = x^0, k = 0,1,2,\cdots\cdots, N-1. \tag{2}$$

其中,$X(k)$是 n维状态向量,$U(k)$为 r维控制向量,$A(k)$和 $B(k)$分别为 $n\times n$和 $n\times r$矩阵,问题为求使目标函数

$$I = X^T(N)Q(N)X(N) + \sum_{k=0}^{N-1}[X^T(k)Q(k) + U^T(k)R(k)U(k)] \tag{3}$$

达到极小值的最优决策 $U^*(0), U^*(1), \cdots\cdots, U^*(N-1)$,其中 $Q(k)(k=0,1,2,\cdots, N-1)$为 $n\times n$正定或半正定对称矩阵;$R(k)(k=0,1,2,\cdots,N-1)$为 $r\times r$正定对称矩阵.

将整个系统分解成 L个子系统,如图 1所示.

$$\begin{array}{ccccccccc}
 & & & U_2 & & & U_L & & \\
 & & & \Downarrow & & & \Downarrow & & \\
U_1 \Rightarrow & S_1 & \overset{X_1}{\Rightarrow} & S_2 & \overset{X_2}{\Rightarrow}\cdots\cdots\overset{X_{L-1}}{\Rightarrow} & & S_L & \overset{X_L}{\Rightarrow} & \\
 & \Downarrow & & \Uparrow & & & \Uparrow & & \\
 & V_1 & & V_2 & & & V_L & &
\end{array}$$

图 1　系统组成

Fig. 1 Conformation of the system

系数矩阵 $A(k)$,$B(k)$分解如下:(为简单省去 k)

$$A = \begin{bmatrix} A_{11} & G_{12} & \cdots \\ G_{21} & A_{22} & \cdots \\ \cdots & \cdots & \cdots \\ G_{L1} & G_{L2} & \cdots & A_{LL} \end{bmatrix};$$

$$B = \begin{bmatrix} B_{11} & F_{12} & \cdots \\ F_{21} & B_{22} & \cdots \\ \cdots & \cdots & \cdots \\ F_{L1} & F_{L2} & \cdots & B_{LL} \end{bmatrix};$$

则分解后的第 i个子系统的状态方程如下:

$$X_i(k+1) = A_{ii}(k)X_i(k) + B_{ii}(k)U_i(k) + V_i(k), \tag{4}$$

$$X_i(0) = X_i^0, \tag{5}$$

$$V_i(k) = \sum_{j=1}^{L} G_j(k)X_j(k) + \sum_{j=1}^{L} F_{ij}(k)U_j(k). \tag{6}$$

第 i个子系统的目标函数:

$$I_i(X_i(k),k) = \min_{U_i(k)}[X_i^T(k)Q_i(k)X_i(k) + U_i^T(k)R_i(k)U_i(k) + I_i(X_i(k+1),k+1)]. \tag{7}$$

整个系统的目标函数为:

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$$I(X(k),k)=\sum_{i=1}^{L} I_i(X_i(k),k). \tag{8}$$

2 引入拉格朗日乘子

要解的问题仍然是使(8)式的值最小,为此引入拉格朗日乘子 $\lambda_i(k)$,

$$\lambda(k)=\sum_{i=1}^{L}\lambda_i(k),\text{令 } h(\lambda)=\min_{X,U,V} f(X,U,V,\lambda,k), \tag{9}$$

$$f(X,U,V,\lambda,k)=\sum_{i=1}^{L}[X_i^T(k)Q_i(k)X_i(k)+U_i^T(k)R_i(k)U_i(k)+I_i(X_i(k+1),K+1)]+\sum_{i=1}^{L}\lambda_i^T(k)[V_i(k)-\sum_{j=1}^{L}G_{ij}(k)X_j(k)-\sum_{j=1}^{L}F_{ij}(k)\cdot U_j(k)]=\sum_{i=1}^{L}\{[X_i^T(k)Q_i(k)X_i(k)+U_i^T(k)\cdot R_i(k)U_i(k)+I_i(X_i(k+1),k+1)]+\lambda_i^T(k)V_i(k)-\sum_{j=1}^{L}\lambda_j^T(k)G_{ij}(k)X_j(k)-\sum_{j=1}^{L}\lambda_j^T(k)F_{ij}(k)U_j(k)\}$$

$$=\sum_{i=1}^{L}f_i(X_i,U_i,V_i,\lambda,k), \tag{10}$$

$$\text{令 } h(\lambda_i(k))=\min_{X_i,U_i,V_i}f_i(X_i,U_i,V_i,\lambda,k). \tag{11}$$

要求的问题是:在(4),(5)两式的约束下,使(11)式达到最小。

注:另一个约束条件(6)式已通过上述拉格朗日函数取消了,此方法的目的就是要把每次计算出的 $V_i(k)$ 和 $\sum_{j=1}^{L}G_{ij}(k)X_j(k)+\sum_{j=1}^{L}F_{ij}(k)U_j(k)$ 的值进行比较,若相等(相等是指两者之差在给定的误差范围内),则输出最优控制 $U_i(k)$,若不相等,继续进行。

3 子系统的动态规划解法

由于所讨论的是离散型线性二次最优控制问题,可以令

$$I_i(X_i(k),k)=X_i^T(k)S_i(k)X_i(k)+P_i(k)X_i(k)+W_i(k). \tag{12}$$

于是

$$I_i(X_i(k+1),k+1)=X_i^T(k+1)S_i(k+1)X_i(k+1)+P_i(k+1)X_i(k+1)+W_i(k+1), \tag{13}$$

将(13),(4)两式依次代入(10)式得(为简单省去 k)

$$f(X,U,V,\lambda,k)=\sum_{i=1}^{L}[X_i^TQ_iX_i+U_i^TR_iU_i+\lambda_i^TV_i-\sum_{j=1}^{L}\lambda_j^TG_{ij}X_i-\sum_{j=1}^{L}\lambda_j^TF_{ij}U_i+X_i^TA_{ii}^TS_i(k+1)A_{ii}X_i+2X_i^TA_{ii}^TS_i(k+1)B_{ii}U_i+2X_i^TA_{ii}^TS_i(k+1)V_i+U_i^TB_{ii}^TS_i(k+1)B_{ii}U_i+2U_i^TB_{ii}^TS_i(k+1)V_i+V_i^TS_i(k+1)V_i+P_i(k+1)A_{ii}X_i+P_i(k+1)B_{ii}U_i+P_i(k+1)V_i+W_i(k+1)], \tag{14}$$

上式分别对 $U_i(k)$ 和 $V_i(k)$ 求一阶偏导

$$\partial f(X,U,V,\lambda,k)/\partial U_i(k)=[2R_i+2B_{ii}^TS_i(k+1)B_{ii}]U_i(k)+2B_{ii}^TS_i(k+1)A_{ii}X_i(k)+2B_{ii}^TS_i(k+1)V_i(k)+B_{ii}^TP_i(k+1)-\sum_{j=1}^{L}\lambda_j^TF_{ij}, \tag{15}$$

$$\partial f(X,U,V,\lambda,k)/\partial V_i(k)=2S_i(k+1)B_{ii}U_i(k)+2S_i(k+1)A_{ii}X_i(k)+2S_i(k+1)V_i(k)+P_i^T(k+1)+\lambda_i(k), \tag{16}$$

令(15)、(16)两式等于零并解方程组,得最优策略和关联向量最优值

$$U_i(k)=M_{i1}(k)X_i(k)+M_{i2}(k), \tag{17}$$

$$V_i(k)=M_{i3}(k)X_i(k)+M_{i4}(k), \tag{18}$$

其中 $M_{i1},M_{i2},M_{i3},M_{i4}$ 分别是 $X_i(k)$ 的系数矩阵和常数项,$U_i(k)$ 就是要求的最优控制策略,$V_i(k)$ 是个协调向量,它表示其它的 $L-1$ 个子系统对第 i 个子系统的关联作用。说明这 L 个子系统不是独立的。但没有把它放在协调级,而是作为计算值出现,这正是与文献[1]的区别所在。

下面推导 $S_i(k),P_i(k),W_i(k)$ 的递推关系式,将(13)、(4)、(17)、(18)各式依次代入(7)式,并与(12)式比较系数可得:

$$S_i(k)=Q_i+M_{i1}^TR_iM_{i1}+A_{ii}^TS_i(k+1)A_{ii}+2A_{ii}^TS_i(k+1)B_{ii}M_{i1}+2A_{ii}^TS_i(k+1)M_{i3}+M_{i1}^TB_{ii}^TS_i(k+1)B_{ii}M_{i1}+2M_{i1}^TB_{ii}^TS_i(k+1)M_{i3}+M_{i3}^TS_i(k+1)M_{i3}, \tag{19}$$

$$P_i(k)=2M_{i2}^TR_iM_{i2}+2M_{i2}^TB_{ii}^TS_i(k+1)A_{ii}+2M_{i4}^TS_i(k+1)A_{ii}+2M_{i2}^TB_{ii}^TS_i(k+1)B_{ii}M_{i1}+2M_{i4}S_i(k+1)B_{ii}M_{i1}+P_i(k+1)B_{ii}M_{i1}+2M_{i2}^TB_{ii}^TS_i(k+1)M_{i3}+2M_{i4}^TS_i(k+1)M_{i3}+P_i(k+1)A_{ii}+P_i(k+1)M_{i3}, \tag{20}$$

$$W_i(k)=M_{i2}^TR_iM_{i2}+M_{i2}^TB_{ii}^TS_i(k+1)B_{ii}M_{i2}+2M_{i2}^TB_{ii}^TS_i(k+1)M_{i4}+M_{i4}^TS_i(k+1)M_{i4}+P_i(k+1)B_{ii}M_{i2}+P_i(k+1)M_{i4}+W_i(k+1). \tag{21}$$

递推的初始条件

$$S_i(N)=Q_i(N);P_i(N)=0;W_i(N)=H_i(N)\text{(任意)}. \tag{22}$$

最优策略由(17)式给出,当初始状态 $X_i(0)=X^0$ 给定时,子系统的目标函数最小值为:

$$I_i^*=I_{i\min}=X_i^{0T}S_i(0)X_i^0+P_i(0)X_i^0+W_i(0).$$

整个系统目标函数最小值为

$$I^*=\sum_{i=1}^{L}I_i^*=\sum_{i=1}^{L}[X_i^{0T}S_i(0)X_i^0+P_i(0)X_i^0+W_i(0)]$$

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