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Almost Periodic Solution of Schoner Models with Diffusion
带扩散 Schoner模型的概周期解

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Abstract The almost periodic solution of non-autonomous diffusion Schoner models is discussed through Liapunov function and differential inequalities. It is found that a unique almost periodic solution exists in that model and remains stable under disturbances from the hull.

Key words diffusion, Schoner model, almost periodic solution, stability under disturbances from the hull

摘要　利用李雅普诺夫函数和微分不等式探讨带扩散 Schoner模型的概周期解的稳定性问题.

关键词　扩散　Schoner模型　概周期解　在壳扰动下的稳定性
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Recently more and more people have been dedicated to the studies of ecosystems for a patch-environment. Schoner(1974) had studied the two species competition system as follows:

$$\begin{cases} \dot{x} = r_1 x\left(\dfrac{I_1}{x+e_1} - r_{11}x - r_{12}y - c_1\right), \\ \dot{y} = r_2 y\left(\dfrac{I_2}{y+e_2} - r_{21}x - r_{22}y - c_2\right), \end{cases}$$

where r_i, I_i, e_i, r_{ij} $(i, j = 1, 2)$ are positive constants. In reference [1] a non-autonomous competition Schoner system with diffusion was studied. Whereas in many circumstances, a few things are truly periodic. So we extend the system in reference [1] to the system with all coefficents which are continuous almost periodic function in this paper. We consider the following system:

$$\begin{cases} \dot{x}_1 = x_1\left[\dfrac{z_1(t)}{x_1+e_1(t)} - r_{11}(t)x_1 - r_{13}(t)y - c_1(t)\right] + D_1(t)(x_2 - x_1) \triangleq f_1(t, x_1, x_2, y), \\ \dot{x}_2 = x_2\left[\dfrac{z_2(t)}{x_2+e_2(t)} - r_{22}(t)x_2 - c_2(t)\right] + D_2(t)(x_1 - x_2) \triangleq f_2(t, x_1, x_2, y), \\ \dot{y} = y\left[\dfrac{z_3(t)}{y+e_3(t)} - r_{31}(t)x_1 - r_{33}(t)y - c_3(t)\right] \triangleq g(t, x_1, x_2, y), \end{cases} \tag{1}$$

where x_i $(i = 1, 2)$ is the density of species x in patch i; y is the density of species y in patch 1; $D_i(t)$ $(i = 1, 2)$ is the diffusion coefficient between patches i and j for species x; $z_i(t), e_i(t), c_i(t), r_{ij}(t), D_i(t)$ $(i, j = 1, 2, 3)$ are continuous and strictly positive almost periodic functions. Now we let $f^u = \sup\limits_{[0,+\infty)} f(t)$, $f^l = \inf\limits_{[0,+\infty)} f(t)$, for a continuous and bounded function $f(t)$. The following arguements are based on the hypothesis that

$$\min\{z_i^l, e_i^l, c_i^l, D_i^l, r_{ij}^l\} > 0, \quad z_i^l - c_i^u e_i^u > 0.$$

1 The Existence and Uniqueness of Almost Periodic Solution

Two lemmas are made before giving main result.

Lemma 1 Suppose system (1) satisfies (H_1), then every solution $\{x_1(t), x_2(t), y(t)\}$ of (1) with positive initial conditions is ultimately bounded in $S = \{(x_1, x_2, y) \mid h \leqslant x_1, x_2, y \leqslant L\}$, namely S is an invariant set of (1), where $h = \min\{m, \bar{y}^l\}$, $L = \max\{M, \bar{y}^u\}$. $m, M, \bar{y}^l, \bar{y}^u$ are the same as in reference [1]. Where

$$\begin{cases} \dfrac{z_3^l - c_3^u e_3^u}{r_{31}^u e_3^u} > \max\left\{\dfrac{z_1^u - c_2^l e_1^l}{r_{11}^l e_3^l}, \dfrac{z_2^u - c_2^l e_2^l}{r_{22}^l e_2^l}\right\}, \\ \dfrac{z_1^l - c_1^u e_1^u}{r_{13}^u e_1^u} > \dfrac{z_3^u - c_3^l e_3^l}{r_{33}^l e_3^l}. \end{cases} \tag{H_1}$$

Proof According to Theorem 3.1 in reference [1] and (H_1), it is easy to learn that the conclusion is correct.

Lemma 2 Suppose system (1) satisfies (H_1) - (H_2), then (1) has a unique solution which is globally attractive.

Where

$$\begin{cases} r_{11}^l > \dfrac{z_1^u}{(e_1^l)^2} + \dfrac{D_2^u}{h} + r_{31}^u, \\ r_{22}^l > \dfrac{z_2^u}{(e_2^l)^2} + \dfrac{D_1^u}{h}, \\ r_{33}^l > \dfrac{z_3^u}{(e_3^l)^2} + r_{13}^u. \end{cases} \tag{H_2}$$

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Proof (H_2) implies (4.1) in reference [1], similar to the proof of Theorem 4.2 in reference [1], we can complete our proof easily.

Theorem 1.1 Suppose system (1) satisfies $(H_1)-(H_2)$, then (1) has a unique almost periodic solution $\{u_1(t),u_2(t),v(t)\}$ which is globally attractive and range $\{u_1(t),u_2(t),v(t)\}\subset S$, $\mathrm{mod}\{u_1(t),u_2(t),v(t)\}\subset \mathrm{mod}\{f_1,f_2,g\}$, for $t\in R$, $(x_1,x_2,y)\in S$.

Proof By the almost periodicity of $z_i(t), e_i(t), c_i(t), r_{ij}(t), D_i(t)$, there exists a sequence of $\{\tau_k\}$, $\tau_k\to+\infty\ (k\to+\infty)$, such that

$$z_i(t+\tau_k)\to z_i(t),\ e_i(t+\tau_k)\to e_i(t),$$
$$c_i(t+\tau_k)\to c_i(t),\ r_{ij}(t+\tau_k)\to r_{ij}(t),$$
$$D_i(t+\tau_k)\to D_i(t),\ i,j=1,2,3 \text{ for all } t\in R. \tag{2}$$

We may suppose $\{\tau_k\}$ is an increase (if necessary, choose subsequence). Hence for any given real number σ there exists $K=K(\sigma)$, such that when $k\geqslant K$, we have $\tau_k+\sigma\geqslant 0$, thus $t+\tau_k\geqslant 0$ for $k\geqslant K, t\geqslant\sigma$. Since S is an invariant set of system (1), for any solution $\{x_1(t),x_2(t),y(t)\}$ of (1), we have

$$\{x_1(0),x_2(0),y(0)\}\in S, t\geqslant\sigma, k\geqslant K\Rightarrow\{x_1(t+\tau_k),x_2(t+\tau_k),y(t+\tau_k)\}\in S.$$

We shall show the function sequence $\{x_1(t+\tau_k), x_2(t+\tau_k), y(t+\tau_k)\}$ is uniformly convergent on each compact subset I of $[\sigma,+\infty)$ as $k\to+\infty$.

Let

$$W(s)=\sum_{i=1}^{2}|\ln x_i(s+\tau_k)-\ln x_i(s+\tau_m)|+|\ln y(s+\tau_k)-\ln y(s+\tau_m)|,\ m\geqslant k\geqslant K, s+\tau_k\geqslant 0, \tag{3}$$

by differential mid-value theorem, we have

$$W(s)\geqslant\frac{1}{L}\{\sum_{i=1}^{2}|x_i(s+\tau_k)-x_i(s+\tau_m)|+|y(s+\tau_k)-y(s+\tau_m)|\}, \tag{4}$$

$$W(s)\leqslant\frac{1}{l}\{\sum_{i=1}^{2}|x_i(s+\tau_k)-x_i(s+\tau_m)|+|y(s+\tau_k)-y(s+\tau_m)|\}. \tag{5}$$

Let

$$\lambda=\min\{r_{11}^l-\frac{z_1^u}{(e_1^l)^2}-\frac{D_2^u}{h}-r_{31}^u, r_{22}^l-\frac{z_2^u}{(e_2^l)^2}-\frac{D_1^u}{h}, r_{33}^l-\frac{z_3^u}{(e_3^l)^2}-r_{13}^u\},$$

clearly $\lambda>0$ by (H_2). For arbitary given $\varepsilon>0$ by (2), there exists a $N=N(\varepsilon,\sigma)\geqslant K$ such that $m\geqslant k\geqslant N, t\in R$, we have

$$\begin{cases}\sum_{i=1}^{3}|c_i(s+\tau_k)-c_i(s+\tau_m)|\leqslant\frac{\lambda h\varepsilon}{12L},\\ \sum_{i=1}^{2}|D_i(s+\tau_k)-D_i(s+\tau_m)|\leqslant\frac{\lambda h\varepsilon}{12L},\\ \sum_{i=1}^{3}\frac{z_i^u}{(e_i^l)^2}|e_i(s+\tau_k)-e_i(s+\tau_m)|\leqslant\frac{\lambda h\varepsilon}{12L},\\ |r_{22}(s+\tau_k)-r_{22}(s+\tau_m)|\leqslant\frac{\lambda h\varepsilon}{12L^2},\\ |r_{11}(s+\tau_k)-r_{11}(s+\tau_m)|+|r_{31}(s+\tau_k)\\ \quad-r_{31}(s+\tau_m)|\leqslant\frac{\lambda h\varepsilon}{12L^2},\\ |r_{33}(s+\tau_k)-r_{33}(s+\tau_m)|+|r_{13}(s+\tau_k)\\ \quad-r_{13}(s+\tau_m)|\leqslant\frac{\lambda h\varepsilon}{12L^2}.\end{cases} \tag{H_3}$$

$$e^{-\lambda h(\sigma+\tau_k)}\leqslant\frac{h\varepsilon}{8L^2}. \tag{6}$$

Then calculating the upper right derivation of $W(s)$ along the solution of system (1), we get

$$D^+W(s)=\mathrm{sgn}[x_1(s+\tau_k)-x_1(s+\tau_m)]\{\frac{x_1'(s+\tau_k)}{x_1(s+\tau_k)}-\frac{x_1'(s+\tau_m)}{x_1(s+\tau_m)}\}+\mathrm{sgn}[x_2(s+\tau_k)-x_2(s+\tau_m)]\{\frac{x_2'(s+\tau_k)}{x_2(s+\tau_k)}-\frac{x_2'(s+\tau_m)}{x_2(s+\tau_m)}\}+\mathrm{sgn}[y(s+\tau_k)-y(s+\tau_m)]\{\frac{y'(s+\tau_k)}{y(s+\tau_k)}-\frac{y'(s+\tau_m)}{y(s+\tau_m)}\}.$$

$$D^+W(s)\leqslant\sum_{i=1}^{3}|c_i(s+\tau_k)-c_i(s+\tau_m)|+\sum_{i=1}^{2}|D_i(s+\tau_k)-D_i(s+\tau_m)|+\sum_{i=1}^{3}\frac{z_i^u}{(e_i^l)^2}|e_i(s+\tau_k)-e_i(s+\tau_m)|+L[|r_{11}(s+\tau_k)-r_{11}(s+\tau_k)|+|r_{31}(s+\tau_k)-r_{31}(s+\tau_k)|]+L|r_{22}(s+\tau_k)-r_{22}(s+\tau_k)|+L[|r_{13}(s+\tau_k)-r_{13}(s+\tau_k)|+|r_{33}(s+\tau_k)-r_{33}(s+\tau_k)|]-\lambda\{\sum_{i=1}^{2}|x_i(s+\tau_k)-x_i(s+\tau_m)|+|y(s+\tau_k)-y(s+\tau_m)|\}\leqslant-\lambda hW(s)+\frac{\lambda h\varepsilon}{2L}. \tag{7}$$

We choose a $N_0\geqslant N$ so that when $t\in I$ and $k\geqslant N_0$, we have $t+\tau_k\geqslant 0$ by using comparison theorem on $[-\tau_k,t]$, thus we obtain

$$W(t)\leqslant W(-\tau_k)e^{-\lambda h(t+\tau_k)}+\frac{\varepsilon}{2L}\leqslant W(-\tau_k)e^{-\lambda h(\sigma+\tau_k)}+\frac{\varepsilon}{2L}. \tag{8}$$

On the other hand, by (4), (5) and the invariant properity of S, we have

$$W(t)\geqslant\frac{1}{L}\{\sum_{i=1}^{2}|x_i(t+\tau_k)-x_i(t+\tau_m)|+|y(t+\tau_k)-y(t+\tau_m)|\},$$

$$W(-\tau_k)\leqslant\frac{1}{h}\{\sum_{i=1}^{2}|x_i(0)-x_i(\tau_m-\tau_k)|+|y(0)-y(\tau_m-\tau_k)|\}\leqslant\frac{4L}{h}.$$

By (8), we get

$$\frac{1}{L}\{\sum_{i=1}^{2}|x_i(t+\tau_k)-x_i(t+\tau_m)|+|y(t+\tau_k)$$

$- y(t+t_m)| \} \leqslant \frac{4L}{h} e^{-\gamma h(\alpha - t_k)} + \frac{\varepsilon}{2L} = \frac{\varepsilon}{L}$,

namely

$$\sum_{i=1}^{2} | x_i(t+t_k) - x_i(t+t_m)| + | y(t+t_k) - y(t+t_m)| < \varepsilon, \quad m \geqslant k \geqslant N_0, t \in I.$$

This implies that $\{x_1(t+t_k), x_2(t+t_k), y(t+t_k)\}$ is uniformly convergent on any compact subset of $[\alpha, +\infty)$ as $k \to +\infty$. Let $\{u_1(t), u_2(t), v(t)\}$ be the limit function of $\{x_1(t+t_k), x_2(t+t_k), y(t+t_k)\}$, since α is arbitarily given, we know that $\{u_1(t), u_2(t), v(t)\}$ is defined on R. Due to range $\{x_1(t), x_2(t), y(t)\} \subset S$ for $t \geqslant 0$, we have range $\{u_1(t), u_2(t), v(t)\} \subset S$.

Similar to the arguement in reference [2], we can prove that $\{u_1(t), u_2(t), v(t)\}$ is differentiable and satisfies system (1).

Similar to the arguement in reference [4], we also can prove that $\{u_1(t), u_2(t), v(t)\}$ is almost periodic and $\mathrm{mod}\{u_1(t), u_2(t), v(t)\} \subset \mathrm{mod}\{f_1, f_2, g\}$.

By Lemma 2.2 we obtain the conclusion that $\{u_1(t), u_2(t), v(t)\}$ is globally attractive with respect to any other solutions of (1) which lies in R_+^3. This complets the proof of Theorem 1.1.

2 Stability under the Disturbances from Hull

Consider any hull system of system (1)

$$\begin{cases} \dot{x}_1 = x_1 \left[\frac{Z_1(t)}{x_1 + E_1(t)} - R_{11}(t)x_1 - R_{13}(t)y - C_1(t)\right] + D_1^*(t)(x_2 - x_1), \\ \dot{x}_2 = x_2 \left[\frac{Z_2(t)}{x_2 + E_2(t)} - R_{22}(t)x_2 - C_2(t)\right] + D_2^*(t)(x_1 - x_2), \\ \dot{y} = y \left[\frac{Z_3(t)}{y + E_3(t)} - R_{31}(t)x_1 - R_{33}(t)y - C_3(t)\right], \end{cases} \quad (9)$$

where

$Z_i(t) \in H(z_i(t)), R_i(t) \in H(r_i(t)), C_i(t) \in H(c_i(t)), E_i(t) \in H(e_i(t)), D_i^*(t) \in H(D_i(t))$.

Theorem 2.1 Assume that the conditions of Theorem 1.1 hold, then every strictly positive solution of (1) (including its unique almost-periodic solution) is stable[2] under disturbances from the hull.

Proof Let $u(t) = \{u_1(t), u_2(t), v(t)\}$ and $x(t) = \{x_1(t), x_2(t), y(t)\}$ be any two strictly positive solutions of (1) and (9) respectively for $t \geqslant t_0$ such that

$h \leqslant u_i(t), v(t) \leqslant L, h \leqslant x_i(t), y(t) \leqslant L, i = 1, 2, \quad x(t_0), u(y_0) \in S, t \geqslant t_0 \quad t_0 \in R$,

now consider a function

$$V(t) = V(u(t), x(t)) = \sum_{i=1}^{2} | \ln u_i(t) - \ln x_i(t)| + | \ln v(t) - \ln y(t)| . \quad (10)$$

It is easy to learn that

$$\frac{1}{L}\left[\sum_{i=1}^{2} | u_i(t) - x_i(t)| + | v(t) - y(t)|\right] \leqslant V(u(t), x(t)) \leqslant \frac{1}{h}\left[\sum_{i=1}^{2} | u_i(t) - x_i(t)| + | v(t) - y(t)|\right]. \quad (11)$$

Calculating the right derivate $D^+ V$ of V, similar to (7) we derive (after simplification) that

$$D^+ V(u(t), x(t)) \leqslant \sum_{i=1}^{3} | c_i(t) - C_i(t)| + \sum_{i=1}^{2} | D_i(t) - D_i^*(t)| + \sum_{i=1}^{3} \frac{z_i^u}{(e_i^l)^2} | e_i(t) - E_i(t)| + L\left[| r_{11}(t) - R_{11}(t)| + | r_{31}(t) - R_{31}(t)| + | r_{22}(t) - R_{22}(t)| + | r_{13}(t) - R_{13}(t)| + | r_{33}(t) - R_{33}(t)|\right] - \gamma\left[\sum_{i=1}^{2} | u_i(t) - x_i(t)| + | v(t) - y(t)|\right], \quad (12)$$

let $P = \max\{\frac{z_i^u}{(e_i^l)^2} : i = 1, 2, 3\}$. An integration of (12) over $[t_0, t]$, with an application of differentiate inequalities leads to (Note that $-\gamma\{\sum_{i=1}^{2} | u_i(t) - x_i(t)| + | v(t) - y(t)|\} \leqslant -\gamma h V(t)$).

$$\frac{1}{L}\sum_{i=1}^{2} | u_i(t) - x_i(t)| + | v(t) - y(t)| \leqslant V(u(t), x(t)) \leqslant V(u(t_0), x(t_0)) + \frac{1}{L}\{\sum_{i=1}^{3} \sup_{t \in R} | c_i(t) - C_i(t)| + P\sum_{i=1}^{3} \sup_{t \in R} | e_i(t) - E_i(t)| + \sum_{i=1}^{2} \sup_{t \in R} | D_i(t) - D_i^*(t)| + L \sup_{t \in R}[| r_{11}(t) - R_{11}(t)| + | r_{31}(t) - R_{31}(t)| + | r_{22}(t) - R_{22}(t)| + | r_{13}(t) - R_{13}(t)| + | r_{33}(t) - R_{33}(t)|]\},$$

and hence

$$\sum_{i=1}^{2} | u_i(t) - x_i(t)| + | v(t) - y(t)| \leqslant \frac{L}{h}\{\sum_{i=1}^{2} | u_i(t_0) - x_i(t_0)| + | v(t_0) - y(t_0)|\} + \frac{L}{\gamma h}\{[\sum_{i=1}^{3} \sup_{t \in R} | c_i(t) - C_i(t)| + P\sum_{i=1}^{3} \sup_{t \in R} | e_i(t) - E_i(t)| + \sum_{i=1}^{2} \sup_{t \in R} | D_i(t) - D_i^*(t)| + L \sup_{t \in R}[| r_{11}(t) - R_{11}(t)| + | r_{31}(t) - R_{31}(t)| + | r_{22}(t) - R_{22}(t)| + | r_{13}(t) - R_{13}(t)| + | r_{33}(t) - R_{33}(t)|]\}.$$

Now for any $\varepsilon > 0$, if we choose $\delta > 0$ such that $\delta < \min\{\frac{\varepsilon h}{5L}, \frac{\varepsilon\gamma h}{5L}, \frac{\varepsilon\gamma h}{5LP}, \frac{\varepsilon\gamma h}{5L^2}\}$, this completes the proof.

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