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Let $A = g + 2d_1x_1^*$, $B = \lambda_1 + 2D_1x_2^*$, $C = \lambda_2 + 2D_2y_2^*$, then $A, B, C > 0$ and

$$|\lambda I - J(E)| = \lambda^3 + (A + B + C)\lambda^2 - \lambda^1\lambda^2 A - gbC + [AB + BC + AC - (\lambda_1\lambda_2 + gb)]\lambda + ABC$$

Noticing $BC > \lambda_1\lambda_2$, then it leads to

$$(A + B + C)[AB + BC + AC - (\lambda_1\lambda_2 + gb)] - (ABC - \lambda_1\lambda_2 A - gbC)$$
$$= 2ABC + \lambda_1\lambda_2 A + gbC - \lambda_1\lambda_2(A + B + C) - gb(A + B + C) + A^2(B + C) + B^2(A + C) + C^2(A + B)$$
$$= 2ABC + BC(BC - \lambda_1\lambda_2) + C(BC - \lambda_1\lambda_2) + A^2(B + C) + B^2A + AC^2 - gb(A + B)$$
$$> ABC + (ABC + A^2B + A^2C + A^2B) - gb(A + B) + AC^2 > (A + B)[A(B + C) - gb] > (A + B)(g\lambda_1 + g\lambda_2 - gb) = g(A + B)(\lambda_1 + \lambda_2 - b) > 0.$$

Therefore, by Routh-Huruitz Theorem, the characteristic roots of $J(E)$ have negative real parts, then there exists a constant $c > 0$ such that all the real parts of characteristic roots are smaller than $-c$. Thus, from reference [6], Theorem 1, system (1) is asymptotically stable on E. Such competes the proof.

We can also get the following Theorem:

Theorem 3 Assume (H_1) holds, then the point $(0, 0, 0)$ is the unstable equilibrium of system (1).

Proof By system (1), the Jacobian matrix on $(0, 0, 0)$ is

$$J(0) = \begin{pmatrix} -g & b & 0 \\ g & -\lambda_1 & \lambda_1 \\ 0 & \lambda_2 & -\lambda_2 \end{pmatrix}.$$

Then its characteristic multinomial of $J(0)$ is

$$F(\lambda) = |\lambda I - J(0)| = (\lambda + g)[(\lambda + \lambda_1)(\lambda + \lambda_2) - \lambda_1\lambda_2] - bg(\lambda + \lambda_2) = \lambda^3 + (g + \lambda_1 + \lambda_2)\lambda^2 + [g(\lambda_1 + \lambda_2) - gb]\lambda - gb\lambda_2.$$

Since $F(0) = -gb\lambda_2 < 0$, $F(+\infty) = +\infty$, there must exist a constant $\lambda^* > 0$ such that $F(\lambda^*) = 0$. Therefore from reference [6], $(0, 0, 0)$ is the unstable equilibrium of the system. Thus completes the proof.

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一类变系数 KdV 型方程的孤立波解

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下列变系数 KdV 型方程描述了一个水深变化的浅水河道中的突变现象[1]

$$u_t + T(t)uu_x + U(t)u_{xxx} = 0, \tag{1}$$

其中 $T(t)$ 和 $U(t)$ 都为可微函数. 在本文中,我们构造 (1) 的如下形式的解

$$u(t,x) = B(t) + A(t)F(s) - A(t)F(s)^2, \tag{2}$$

其中 $F(s) = 1/(1 + \exp s)$, $s(t,x)$, $B(t)$ 和 $A(t)$ 为待定函数,并且 $s_{xx} = 0$. 把 (2) 代入 (1),比较 F^j 的系数,我们可以得到:

$$F^5:\ 2A^2s_xT - 24As_x^3U = 0; \tag{3}$$
$$F^4:\ 60As_x^3U - 5A^2s_xT = 0; \tag{4}$$
$$F^3:\ -50As_x^3U + (4A^2 - 2BA)s_xT - 2As_t = 0; \tag{5}$$
$$F^2:\ 15As_x^3U - (A^2 - 3BA)s_xT + 3As_t - A_t = 0; \tag{6}$$
$$F^1:\ As_x^3U + BAs_xT + As_t - A_t = 0; \tag{7}$$
$$F^0:\ B_t = 0. \tag{8}$$

由式 (3) ～ (8) 我们得到 B 为一个任意常数,并且

$$A = 12\frac{U(t)}{T(t)}s_x^2,\ A_t = 0,\ s_t = -Us_x^3 - BTs_x, \tag{9}$$

由此得到 $U(t) = lT(t)$,其中 l 为一个任意常数. 由 (9) 和 $s_{xx} = 0$,我们可以得到

$$s(t,x) = kx - (k^3l + kB)\int T(t)\,dt, \tag{10}$$

其中 k 为一个非零常数. 容易证明 $F(s) - F(s)^2 = (1/4)\operatorname{sech}^2(s/2)$. 因而,如果满足条件 $U(t) = lT(t)$,则方程 (1) 有如下孤立波解

$$u(t,x) = B + 3lk^2\operatorname{sec}h^2\left(\frac{1}{2}s\right), \tag{11}$$

其中 s 如 (10) 所示. 文献 [1] 中的结果不包含解 (11).

我们发现,这种直接方法还可以应用于许多变系数的反应扩散方程的求解.

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