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Targeting and Controlling Instabilities of Circle Map
圆映象不稳定性的控制与瞄准

Liu Zonghua* Chen Guangzhi Ni Wansun
刘宗华 陈光旨 倪皖荪

(Department of Physics, Guangxi University, 10 Xixiangtanglu, Nanning, Guangxi, 530004)
(广西大学物理系 南宁市 西乡塘路 10号 530004)

(Institute of Acoustics, Nanjing University, Hankoulu, Nanjing, Jiangsu, 210008)
(南京大学声学研究所 江苏省南京市汉口路 210008)

Abstract The control of unstable periodic orbit of Circle map was implemented by adjusting parameter and changing Lyapunov exponent of the system. When the rotation number was irrational, we implemented the target from one point of quasiperiodic orbit to another point on the same quasiperiodic orbit. This kind of controlling and targeting has a power of resistance to noise.

Key words control, rotation number, quasiperiodic orbit

摘要 通过调节参数与改变系统的 Lyapunov 指数，实现了园映象失稳周期轨道的控制。当旋转数为无理数时，实现了从准周期轨道上一点到另一点的瞄准。这种控制与瞄准具有一定的抗干扰能力。

关键词 控制 旋转数 准周期轨道
中图法分类号 O414

The chaos is ever thought to be harmful because of its irregularity and uncontrollability. But now people have changed their mind. One can control the chaotic system to an originally unstable orbit or to a object point by using of the sensitivity of chaos to initial point and the ergodicity. In all of the methods, the E. Ott, C. Grebogi and J. A. Yorke's (O. G. Y) method is the best one[1~3], which can be implemented by experiment[4~5] because of its small perturbation to a system parameter. Ni Wansun et al developed this method into bifurcation region[6]. Furthermore, we also use this method to direct trajectory to target[7]. In this paper, we first develop this method to control the unstable orbit of Sine-Circle map, then we give a new method to implement the target of quasiperiodic orbit. Because the rotation number of quasiperiodic orbit is irrational, the properties of the quasiperiod can not be kept if we change the parameter of the system, so the target of quasiperiodic orbit must be kept the rotational number irrational by using some new method. This paper is to discuss this case including targeting the non-quasiperiodic orbit of Circle map.

It is known that Circle mappings are the simplest models of coupled nonlinear oscillators. They show a variety of new phenomena, e. g., quasiperiodic motion, mode-locking and transitions from quasiperiodic motions to chaos. Hence we use the Sine-Circle map as an example in this paper, in which noise is also considered.

This paper is organized as follows. In Sec. 2 we control the unstable periodicorbit. Then in Sec. 3 we give a new method to implement the target of quasiperiodic orbit. In Sec. 4 we discuss the target of any point and the effect of noise.

1 Stabilizing unstable periodic orbit in circle map

Consider following Sine-Circle map

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* 中国工程物理研究院研究生部 北京 100088 (Graduate School, China Academy of Engineering Physics, P. O. Box 2101, Beijing, 100088).

$$\theta_{n+1} = f(\theta_n) = \theta_n + \text{k} - \sin(2C\theta_n)\kappa/2C \pmod{1} \quad (1)$$

The rotation number is

$$\Omega = \lim_{n\to\infty}(f^{(n)}(\theta) - \theta)/n \pmod{1} \quad (2)$$

Let k= 0. 6, κ = 0– – 3, we can get the bifurcation diagram as fig. 1

From fig. 1, we find that there are the orbit of period–5 near κ = 1. 2, the orbit of period–3 near κ = 1. 5, the orbit of period–2 near κ = 1. 8, and a quasiperiodic state near κ = 0. 5. These stable orbits will become unstable with changing parameter κ.

The method to control periodic orbits of period n has been given in ref [2]. It can be applied in our case. For example, if there are unstable periodic points of period–3, $\theta_1^*, \theta_2^*, \theta_3^*$, and θ_{n+i} is in the neighborhood of θ_i^* (i = 1, 2, 3), then we can control it as follows

$$\kappa_{n+i} = \bar{\kappa} - \text{X}(\theta_{n+i} - \theta_i^*)$$

$$\theta_{n+i+1} = f(\theta_{n+i}, \kappa_{n+i})\ (i = 1, 2, 3) \quad (3)$$

By use of $\theta_{i+1}^* = f(\theta_1^*, \bar{\kappa})$ and eqs. (3), one has

$$\theta_{n+i+1} - \theta_{i+1}^* = f(\theta_{n+i}, \kappa_{n+i}) - f(\theta_i^*, \bar{\kappa}) = (f_\theta - \text{X}f_\kappa)_{\theta=\theta_i^*, \kappa=\bar{\kappa}}(\theta_{n+i} - \theta_i^*) + \cdots \quad (4)$$

It is worthy of noting that $\theta_{3+i}^* = \theta_i^*$ since θ_i^* (i = 1, 2, 3) are the unstable periodic points of period–3. From (4) the condition of convergence of iteration is

$$|f_\theta - \text{X}f_\kappa| < 1 \quad (5)$$

The condition of optimum control is

$$f_\theta - \text{X}f_\kappa = 0$$

$$\text{X} = (\text{X})_{op} = \frac{f_\theta}{f_\kappa} = \frac{1 - \kappa\cos(2C\theta_i^*)}{-\sin(2C\theta_i^*)/(2C)} \quad (6)$$

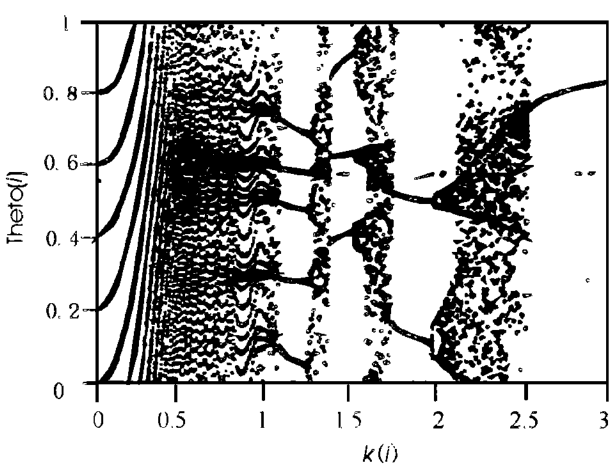

Fig. 1 Bifurcation diagram for k = 0. 6, κ = 0– – 3

Obviously the above method of control of unstable periodic orbit of period–3 can be extended to that of period N (N is any interger number), which has been noticed in ref. [6]. This method can be applied not only to chaotic attractor but also to the case of bifurcation if both condition (5) and some other condition (see condition (6) in ref. [6]) are satisfied.

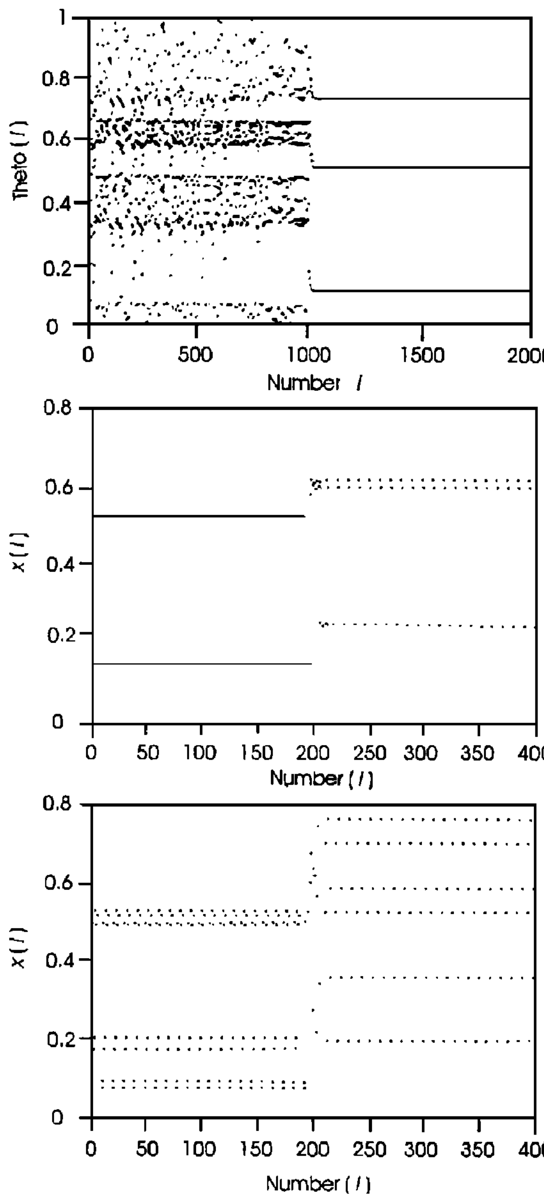

Fig. 2 Shown the control of unstable periodic orbit (a) Control of unstable period–3 in chaotic region at k = ($\sqrt{5}$ – 1) /2, and κ = 1. 5, the first 1000 points denote chaotic motion. Control begins at n = 1001; (b) Control of unstable period–3 in bifurcation region at k = 0. 6, and κ = 1. 8 where period 2 is stable. (c) Control of unstable period 6 in bifurcation region of period 8 at k = 0. 6, and κ = 2. 0 where period 8 is stable.

Some numerical examples are shown in fig. (2) (a) -(c). (a) denotes the control of unstable period-3 in chaotic region at $k = (\sqrt{5} - 1)/2$, and $\kappa = 1.5$, where the first 1000 points are chaotic motion, and control begins at $n = 1001$; (b) denotes the control of unstable period-3 in bifurcation region at $k = 0.6$, and $\kappa = 1.8$ where period-2 is stable, then it needs only 6 steps to implement the control; and (c) denotes the control of unstable period-6 in bifurcation region $k = 0.6$, and $\kappa = 2.0$ where period-8 is stable, then it needs about 10 steps to implement the control.

2 Targeting quasiperiodic orbit

Although we had given a general targeting method by parameter adjustment, the method does not fit the target of quasiperiodic orbit. It is because we must keep the rotation number as irrational in the case of targeting quasiperiodic orbit. We can not keep the rotation number as irrational if we change the parameter of the system, so we need a new method for implementing the target of quasiperiodic orbit.

Since the rotation number of quasiperiodic orbit is irrational in this paper, we let the rotation number equal to golden number $\Omega = (\sqrt{5} - 1)/2$. There are two cases. One is that we keep Ω as constant in the targeting process, that is targeting from one point to another point of the same quasiperiodic orbit. This case is difficult to be implemented because the rotation number will become rational if we do not have enough precaution in parameter adjusting process. The other case is targeting from one point of quasiperiodic orbit to any point which may not be on the quasiperiodic orbit or from any point to one point of quasiperiodic orbit. To the former, there is some relation between k and κ because we have to keep the same irrational number Ω at both initial point and object's point. In this paper, let $\kappa = 0.3$, then k must be about 0.616543 in order to keep $\Omega = (\sqrt{5} - 1)/2$. Our targeting method is as follows. Firstly, we let the system start from the initial point and go into the neighbourhood of target point by iterating without controlling. We can measure the error of θ_N to object $\triangle = \theta_N - \theta_{object}$ if this process need N steps. Secondly, we feedback this error to the N steps of iteration process, that is we reiterate the N steps by adding feedback quantity. If θ_N will be nearer to the object, then error $\triangle$ will become smaller. Repeating this process, we can precisely target the object.

Detailed process of targeting is as follows: we change the θ_n in (1) in every step by δ, then

$$\theta_{n-new} - \theta_{n-old} = W - \kappa\cos(2\pi\theta_n)W = W(1 - \kappa\cos(2\pi\theta_n)) \quad (7)$$

Then we let the sum of changed value in N-steps be $\triangle$, that is

$$\sum_{i=1}^{N} W(1 - \kappa\cos(2\pi\theta_i)) = \triangle = \theta_N - \theta_{object}$$

$$W = \frac{\triangle}{\sum_{i=1}^{N}(1 - \kappa\cos(2\pi\theta_i))} \quad (8)$$

Hence we get the feedback quantity δ from error $\triangle$. Now we use this feedback method to implement the target. Firstly, we iterate the N steps again by $h\theta_{n+1} = f(\theta_n + W)h$, then it must target the object more accurate than purely iteration (1). Then we measure the new error between θ_N and θ_{object}, we can get new W' by (8). Using this new W' and repeating this process, we can let the error between θ_N and object becomes smaller. If we repeat this process several times, we can target the object in any precision. For example, the rotation number of the quasiperiodic orbit of circle map is $\Omega = (\sqrt{5} - 1)/2$ when $k = 0.616543$, and $\kappa = 0.3$. We target from $\theta_0 = 0.9$ to $\theta_{object} = 0.3$ on the same quasiperiodic orbit without changing k and κ. Firstly we iterate (1) without any controlling, we find $f^{(7)}(\theta_0) = 0.26876$, $\triangle = 0.26876 - 0.3 = -0.03124$ and $W = -0.0038465$; then we implement targeting by use of $f(\theta_n + W)$ and get $f^{(7)}(\theta_0 + W) = 0.2964$. The new $\triangle = 0.2964 - 0.3 = -0.0036$ and the new $W' = -0.004289$. Target again with $f(\theta_0 + W + W')$, and we get $f^{(7)}(\theta_0 + W + W') = 0.2996$. This value is very close to the θ_{object}, So we hit the target $\theta_{object} = 0.3$ in the range of approximation.

The above method is very useful in targeting. But how can one change θ_i into $\theta_i + W\theta$ in experiment? The answer is to change parameter k by instantaneous perturbation. For example, if we use $k + Wk$ instead of k in eq. (1), θ_{i+1} will become $\theta_{i+1} + W\theta$. Then we reture k to original value so that we can keep the rotation number as irrational. Continuing this process, we can implement the above idea in experiment.

3 Targeting of any point

Now we discuss another case of targeting, i. e. , when we direct the trajectory from the initial point θ_i to the target point θ_{object} , the rotation number Ω can be changed. In this case, the feedback method of targeting described in ref. [7] can be applied. From ref. [7], the metod is as follows.

When x_0 is at initial point and $x_{n+1} = f^{(m)}(x_0, p_n)$ is in the neighbourhood of target point x_t, p is an adjustable parameter, then we can use the following method for targeting

$$x_{n+1} = f^{(m)}(x_0, p_n)$$

$$p_{n+1} = p_n - X(x_{n+1} - x_t) \quad (9)$$

The condition of convergence is $|\lambda| < 1$, here

$$\lambda = \lim_{n\to\infty} \frac{x_{n+1} - x_t}{x_n - x_t} = 1 - X f_p^{(m)} \quad (10)$$

But now we give a more effective method. This new method combines the adjustment of parameter and feedback to implement targeting. The targeting steps of this method are about 10 steps, which are less than those of the parameter adjusting or feedback. This method is as follows:

Considering 1-D map $x_{n+1} = f(x_n, k_n)$, then

$$x_{n+1} - x_n = f(x_n, k_n) - f(x_{n-1}, k_{n-1})$$
$$= f_x(x_n - x_{n-1}) + f_k(k_n - k_{n-1}) + \cdots \quad (11)$$

$$x_{n+1} - x^* = x_n - x^* + f_x(x_n - x_{n-1}) + f_k(k_n - k_{n+1}) + \cdots$$

Let $x_n - x_{n-1} = -X_1(x_n - x^*)$

$$k_n - k_{n-1} = -X_2(x_n - x^*) \quad (12)$$

and neglect high-order items of (8) in the linear approximation, we have

$$x_{n+1} - x^* = x_n - x^* - X_1 f_x(x_n - x^*) - X_2 f_k(x_n - x^*)$$

$$(x_{n+1} - x^*)/(x_n - x^*) = 1 - X_1 f_x - X_2 f_k \quad (13)$$

The condition of convergence of iteration is that we have to choose someproper control stiffness X_1 and X_2 , so that $|1 - X_1 f_x - X_2 f_k| < 1$, then the iterating series x_n will approach object. For example, for the Sine-Circle map, we let

$$X_1 = \begin{cases} 0 & |\theta_i| < 0.001 \\ -\dfrac{2c(1 - \kappa_i \cos(2c\theta_i))}{\sin(2c\theta_i)} & |\theta_i| > 0.001 \end{cases}$$

$$X_2 = 0.3 \quad (14)$$

Some numerical example are shown in fig. 3.

Fig. 3 (a) denotes the targeting from one point of quasiperiodic orbit for $k = 0.616543$, $\kappa = 0.3$ to period 3 for $k = 0.66$, $\kappa = 0.8$, where the first 100 points denote quasiperiodic motion. Targeting begins when $n > 100$. It needs 12 steps to implement the targeting. Fig. 3 (b) denotes the targeting from one point of quasiperiodic orbit for $k = 0.616543$, $\kappa = 0.3$ to point $\theta_{object} = 0.8$, which needs 2 steps. The first 300 points denote quasiperiodic motion. Targeting begins at the 301th point.

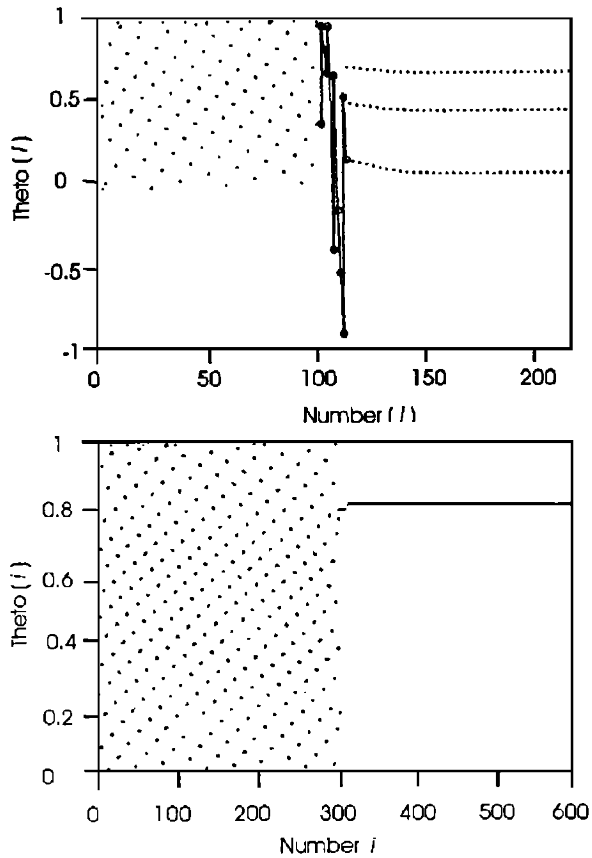

Fig. 3 Shown the targeting of circle map
(a) From one point of quasiperiodic orbit for $k = 0.616543$, and $\kappa = 0.3$ to period-3 for $k = 0.66$, and $\kappa = 0.8$, the first 100 points are quasiperiodic motion. Targeting begins when $n > 100$. It needs 12 steps to implement the targeting; (b) From one point of quasiperiodic orbit for $k = 0.616543$, and $\kappa = 0.3$ to point $\theta_{object} = 0.8$, it needs 2 steps, the first 300 points denote quasiperiodic motion, targeting begins at the 301th point.

There is always external noise in real system. Inorder to check the stability of above method when noise is present, we add white noise to the system. The intensity of noise is about 5.8×10^{-3}, and the control results are shown in fig. 4

Comparing fig. 4 (a) with fig. 2 (a), and fig. 4 (b) with fig. 3(a), we see that both controlling and targeting are achievable.

In conclusion, we have implemented the controlling of Circle map. Moreover, we have given two kind of targeting method, one for the quasiperiodic orbit, another for the general case. Particularly, we should point out that the targeting of quasiperiodic orbit is very interesting. These methods are able to resist noise, too.

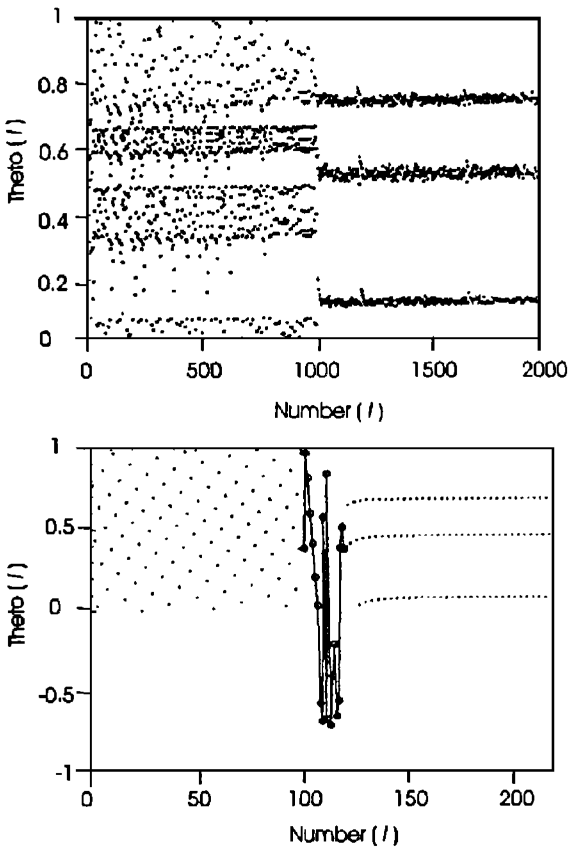

Fig. 4 Shown adding noise case

(a) The control of unstable period-3 in chaotic region for k= ($\sqrt{5}$ - 1) /2, and κ = 1. 5, the first 1000 points denote chaotic motion. Control begins at the 1001th point. (b) targeting from quasiperidic orbit for k= 0. 616543, and κ = 0. 3 to any point for k= 0. 66, and κ = 0. 8, which needs 20 steps. The first 100 points denote quasiperiodic motion. Targeting begins at the 101th point.

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