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等径、对称叉管展开的数解法

Mathematic Analytic Method for Development of the Forked Pipe with Equal Diameter and Symmetry

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摘要 对斜椭圆柱面叉管的展开提供了一套简便的计算方法；通过分析，推导出叉管展开的一套简单、通用的解析公式。

关键词 近似解法 数解法 等径、对称叉管

Abstract A series of short-out methods of calculation for development of the forked pipe of the oblique elliptic cylinder are presented, and a series of simple and general analytic formulas for development of the forked pipe are derived from analysis.

Key words approximate analytic method, mathematic analysis, forked pipe with equal diameter and symmetry

The forked pipe of the oblique elliptic cylinder are widely used in the hydroelectric project. Before making steel forked pipe it is necessary to draw a developed view for the forked pipe firstly. Forked pipe may be developed by two methods: graphics and mathematic analysis. The graphic method is difficulty in drawing and not accurate owing to the fact that the pipe-piece is bigger in diameter, especially, the elliptic pipe-piece is more complicated in drawing. Thereby an approximate drawing method is often used in practical operation, but it causes a certain error. Although a semi-calculation and semi-graphical method was intruduced in related informations, it is more complex and redundant. This paper mainly intruduces a series of more simplified and general analytic formulas for development of the forked pipe of the oblique elliptic cylinder.

1 Calculation of elliptic arc length

A standard ellipse is shown as in Fig. 1. Suppose the major axis of the ellipse is a, the minor axis is b. The point T is the centre of a circle. using a and b as a radius to draw concentric circles, respectively, known as auxiliary circles. The auxiliary circumferences are evenly divided in M divisions. (In Fig. 1. $M=12$).

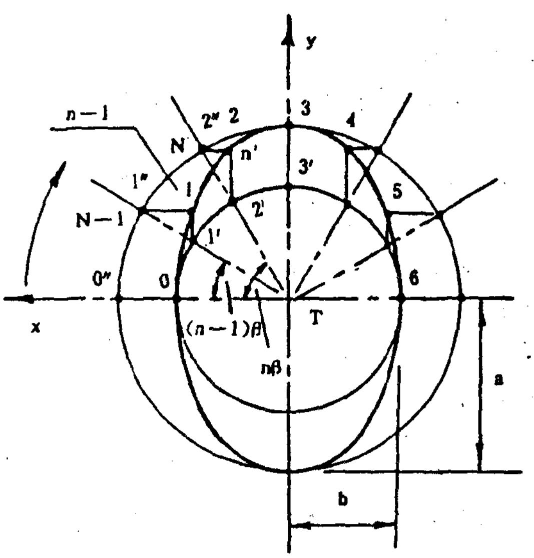

Fig. 1

According to the graphic method, when the workpiece is developing, its normal section or auxiliary circumferences may be evenly divided into M divisions.

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Consider the symmetry of drawing by the graphic method and the convenience of developing calculation by mathematic analysis, and make each division of the circular arc length or the elliptic arc length corresponding to it not too long, so as to meet the need of the accuracy in the project. By actual developing calculation and lofting in operation the table of developed circle divisions (see table 1) is recommended.

The equiangular division corresponding to evenly-divided circular arc is expressed as β, yield:

$$\beta = \frac{360°}{M} \qquad (1)$$

In Fig. 1, suppose N and $(N-1)$ are two adjacent moving ones among the points of the auxiliary circumference, the central angles corresponding to them are $n\beta$ and $(n-1)\beta$, respectively, and their correspondent points on elliptic circle are n and $(n-1)$, respectively. Suppose the coordinates of the point n are (Xn, Yn), the ones of the point $(n-1)$ are $(Xn-1, Yn-1)$, From the parameter equation of the standard ellipes, yield:

$$\begin{cases} Xn = b\cos n\beta \\ Yn = a\sin n\beta \end{cases}$$

$$\begin{cases} Xn-1 = b\cos(n-1)\beta \\ Yn-1 = a\sin(n-1)\beta \end{cases}$$

Table 1 The table of developed circle divisions

Diameter D (mm)	Number of Divisions M	Equiangular Divisions β (Degree)
100 (below)	8 (used less)	45°
100～250	12	30°
250～500	16	22.5° (22°30')
500～750	24	15°
750～1000	32	11.25° (11°15')
1000～1500	48	7.5° (7°30')
1500～2000	72	5°
2000 (over)	96	3.75° (3°45')

The arc length between the two adjacent points on the elliptic circle may be approximately replaced by the chord length, and marked by $\triangle Sn(n-1)$. From the equation of the distance between the two points, yield:

$$\triangle Sn(n-1) = \sqrt{(Xn - Xn-1)^2 + (Yn - Yn-1)^2}$$
$$= \sqrt{b^2[\cos n\beta - \cos(n-1)\beta]^2 + a^2[\sin n\beta - \sin(n-1)\beta]^2}$$

After arranging, found:

$$\triangle Sn(n-1) = a\sqrt{k_1 - k_2(k_3\cos n\beta - \sin n\beta)^2} \qquad (2)$$

Where:

$$\left.\begin{aligned} k_1 &= 2(1-\cos\beta) \\ k_2 &= [1 - \frac{b}{a}^2]\sin^2\beta \\ k_3 &= \frac{1-\cos\beta}{\sin\beta} \end{aligned}\right\} \text{coefficient}$$

$$n = 0, 1, 2, \cdots, M$$

Suppose the perimeter of the ellipse is s, yield:

$$S = \sum_{n=1}^{M} \triangle Sn(n-1) \qquad (3)$$

2 The Development of an oblique elliptic cylinder

The developed view of the oblique elliptic cylinder is shown as in Fig. 2. The prime line of the oblique elliptic cylinder is parallel to orthographic projection section. Its front projection reflects the real length. Its normal section is an ellipse. Its horizontal section is also an ellipse in a general way. but when the angle α between the cylinder axis and the horizontal axis be conformed that $\sin\alpha = b/a$, the horizontal section is a circle, its radius $R = a$, The horizontal section of the oblique elliptic cylinder used in the hydroelectric project is generally designed to be a circle.

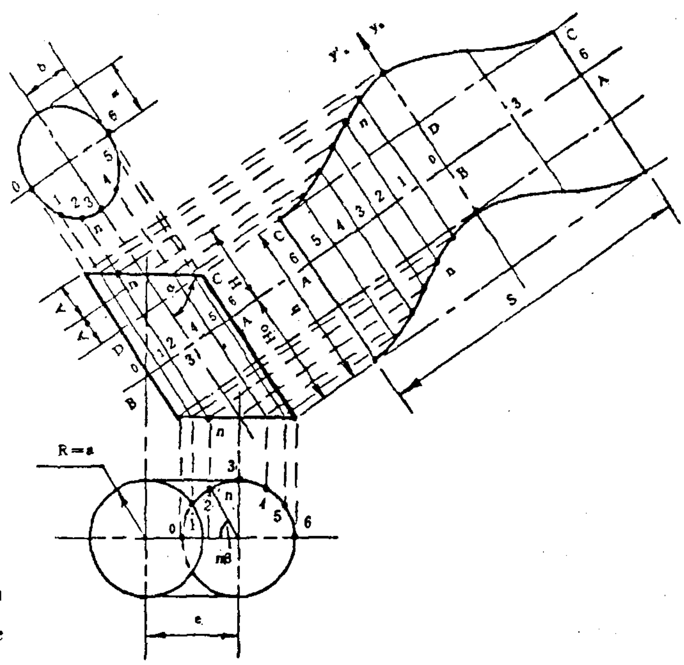

Fig. 2

In Fig. 2 the developed view of the oblique elliptic cylinder is equivalent to a rectangular section on which a camber at the lower extreme is cut and transplanted to the upper extreme. the height of the rectangular section is the real length of the prime line of the oblique elliptic cylinder (All the prime lines are the same in length). the length of the bottom side of the rectangular section is equal to the developed perimeter S of the ellipse. The arc length between the two adjacent points on the elliptic circle is equal to the line segment's length between the two points corresponding to the bottom side of the rectangular section.

In Fig. 2. if granted R, e and α, following steps may be adapted in its calculation of development.

Calculation of basic data

$$A = R\cos\alpha \qquad (4)$$

$$h = \frac{e}{\cos\alpha} \qquad (5)$$

Calculation of development

According to the size of the level circle' s diameter $D = 2R = 2a$ of the oblique elliptic cylinder, proper M value is selected from table 1, the level circumference is evenly divided into M divisions, so the ellipse perimeter, the oblique elliptic cylinder and bottom side of its developed view will be sure to divide into correspondent divisions (but not even). As a result of the pattern's symmetry, developing calculation just needs for a half only.

The abscissas of the various points on the developed curve.

From Fig. 2, $a = R, b = R\sin\alpha$, $\sin\alpha = b/a$, so the expression (2) and (3) may be changed into.

$$\triangle Sn(n-1) = R\sqrt{k_1 - k_2(k_3\cos n\beta - \sin n\beta)^2} \qquad (6)$$

Where:

$$\left.\begin{aligned} k_1 &= 2(1-\cos\beta) \\ k_2 &= [1-(b/a)^2]\sin^2\beta = (\cos\alpha\cdot\sin\beta) \\ k_3 &= \frac{1-\cos\beta}{\sin\beta}) \end{aligned}\right\} \text{coefficient}$$

$$n = 0,1,2,\cdots,M/2$$

$$Xn = \sum_{n=1}^{M/2}\triangle Sn(n-1) \qquad (7)$$

The ordinates of the various points on the developed curve (take abscissa axis CD as starting calculating datum):

From Fig. 2, known:

$$A = R\cos\alpha$$

$$Yn = A + R\cos n\beta\cdot\cos\alpha$$

$$\therefore Yn = A(1+\cos n\beta)$$

Where:

$$n = 0,1,2,\cdots,M/2$$

Owing to the need of development of the following forked pipe, if the abscissa axis CD is parallelly translated to the line AB, and value H is known (see Fig. 2), value Yn can then be rewritten as

$$Yn' = H + R\cos n\beta\cdot\cos\alpha$$

$$\therefore Yn' = B + A\cos n\beta$$

$$\text{and } H_0 = h + A - H$$

3 Development of the two-forked pipe of the oblique elliptic cylinder

The two-forked pipe of the oblique elliptic cylinder is shown as in Fig. 3. If R, e and α_m are known, these various pipe-pieces be respectviely developed.

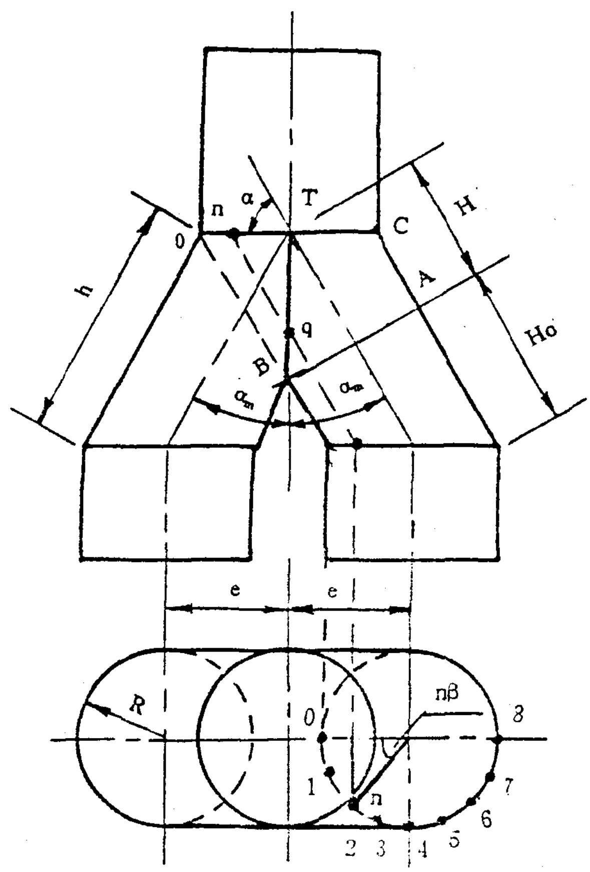

Fig. 3

From Fig. 3 it may be seen that main and branch pipes are all right cylinder ones, their radii are R and the developed views are all rectangular seetions on which the bottom sides are $2\pi R$ (developed view is omitted).

The forked pipe, in fact, consists of two oblique elliptic column pipes with a nook cut. The front projection of their intersection line is line TB. This is greatly convenient of developing calculation.

From Fig. 3 it can be seen that

$$\alpha = 90°\alpha_m \qquad (11)$$

$$B = R\operatorname{ctg}\alpha_m\cdot\cos\alpha_m \qquad (12)$$

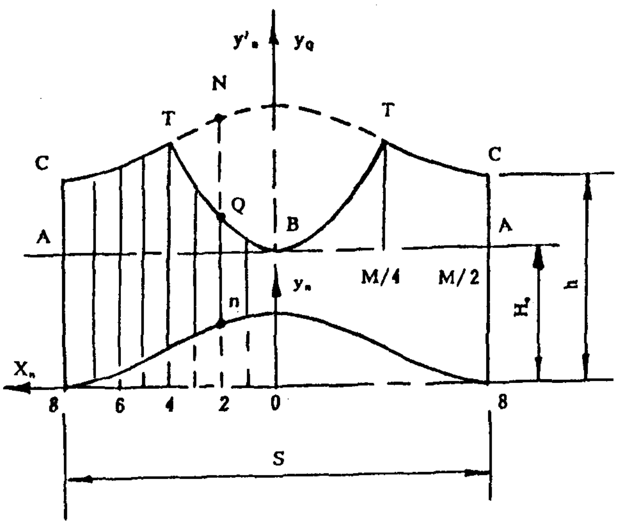

Fig. 4

The developed view of the two-forked pipe of the oblique elliptic cylinder is shown as in Fig. 4. In Fig. 3, passing the end point B of the intersection line TB to draw a normal section AB of the forked pipe and divide it into upper and lower parts. The calculation of development of the lower extreme may be conducted by the expression (8)and, at upper extreme, the calculation of development of the intersection line TC by the expression (9), in order to integrate the coordinates, but it is necessary that the value n in the expression (9) should be ordered to take $n = M/4, \cdots, M/2$.

Calculation of development of the intersecction line TB is as follows.

Suppose nq is the length of cut-parts of any prime line on the front vertical projective section of the forked pipe. NQ is the corresponding line segment of nq in the developed view (see Fig. 4) Owing to front vertical projective section, its front projection reflects the real length, hence $nq = NQ$.

In Fig. 3, from the nature of similar triangles, it can be found that

$$\frac{nq}{OB} = \frac{Tn}{TO} \quad i.e. \quad \frac{nq}{R/\sin\alpha_m} = \frac{R\cos n\beta}{R}$$

$$\therefore nq = \frac{R\cos\beta}{\sin\alpha_m}$$

order: $k_1 = \dfrac{R}{\sin\alpha_m}$ (coefficient)

yield: $NQ = k_1 \cos n\beta$ (13)

From Fig. 4, the ordinate value of the point Q can be found, as

$$YQ = Yn' - NQ,$$

Substituting into the expression (9) and (13), yields

$$YQ = H(1 - \cos n\beta) \quad (14)$$

Where:

$n = o, 1, 2, \cdots, M/4$

4 Calculation example of development of the forked pipe

The water-transferred forked pipe of an oblique elliptic cylinder is shown as Fig. 3. Given $R = 240$mm, $\alpha_m = 30°$, $e = 340$mm, calculate the forked pipe's development.

Main and branch ones are all right column pipes in radii as R. Their developed views are all rectangles with bottom sides as $2\pi R$. (Developed view is omitted).

Following calculation of development is just for forked pipe, and it needs for half developing calculation only because of the pattern's symmetry. Developed view of the forked pipe is shown as Fig. 4.

4.1 Selection of value M

From the diameter of the main pipe $D = 2R = 240$mm, look up table 1. and yields, $M = 16$, $\beta = 22° 30'$.

4.2 Calculation of basic data

Substituting some data known into expression (11), (5), (4), (12) and (10), yields

$\alpha = 90° - \alpha_m = 60°$

$h = \dfrac{e}{\cos\alpha} = 680$mm

$A = R\cos\alpha = 120$mm

$H = R\,\mathrm{ctg}\alpha_m \cdot \cos\alpha_m = 360$mm

$H_0 = h + A - H = 440$mm

$k_1 = 2(1 - \cos\beta) = 0.15224$

$k_2 = (\cos\alpha\cos\beta) = 0.036661$

$k_3 = \dfrac{(1-\cos\beta)}{\sin\beta} = 0.19891$

4.3 Calculation of the abscissa

Calculating by the expression (6) and (7)

$$\triangle Sn(n-1) = R\sqrt{k_1 - k_2(k_3\cos n\beta - \sin n\beta)^2}$$

$$= 240\sqrt{0.15224 - 0.03661(0.19891\cos n\beta - \sin n\beta)^2}$$

where:

$n = 0, 1, 2, \cdots, 8$

$$Xn = \sum_{n=1}^{8} \triangle Sn(n-1)$$

Developed calculations (including all the calculations below) are listed on Table 2.

4.4 Calculation of the ordinate on the developed curve of the forked pipe's lower extreme

Calculating by expression (8)

$Yn = A(1+\cos n\beta) = 120(1+\cos n\beta)$

where:

$n = 0, 1, 2, \cdots, 8$

4.5 Calculation of the ordinate on the developed curve of the forked pipe's upper extreme

Calculating by the expression (14) and (9)

$YQ = H(1-\cos n\beta) = 360(1-\cos n\beta)$

where:

$n = 0, 1, \cdots, 4$

$Yn' = H + A\cos n\beta = 360 + 120\cos n\beta$

where:

$n = 4, 5, \cdots, 8$

4.6 Arrangement of the fruits

Finally, the value n, x and y in table 2 are listed and arranged reasonably in a drawing sheet with Fig. 3 and Fig. 4. that is the fruits of the forked pipe's design.

5 Conclussion

The methods of development of the forked pipe of the oblique elliptic cylinder and main formulas of calculation in this paper can be used to calculate the development of other pipe-pieces, such as the elliptic cone-face. And in practical operation it can be satisfied with accuracy of practice and, simple, covenient and easy to be applied.

Table 2

n	$n\beta$	$\cos n\beta$	$\sin n\beta$	$\triangle Sn(n-1)$	Xn	Yn	YQ	Yn'
0	0°	1	0		0	240.00	0	
1	22°30'	0.92388	0.38268	93.20	93.20	230.87	27.40	
2	45°	0.70711	0.70711	89.96	183.16	204.85	105.44	
3	67°30'	0.38268	0.92388	85.17	268.33	165.92	222.24	
4	90°	0	1	81.61	349.94	120.00	360.00	360.00
5	112°30'	−0.38268	0.92388	81.61	431.55	74.08		314.08
6	135°	−0.70711	0.70711	85.17	516.72	35.15		275.15
7	157°30'	−0.92388	0.38268	89.96	606.68	9.13		249.13
8	180°	−1	0	93.20	699.88	0		240.00

References

1 武汉水利电力学院. 工程曲面的几何计算与展开. 北京：水利出版社，1980.

2 郑灿新. 叉管展开的数解法. 武汉水利电力学院学报，1984，(1).

3 郑灿新. 正圆柱面叉管展开的数解法. 广西机械，1988，(3)，(4).